Heuristics for the Generalized Traveling Salesman Problem

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Introduction

- Many real-world problems involve optimization.
- Some problems are relatively easy for computers to solve optimally.
  - Shortest path is an example.
- Other problems are much harder to solve with large input.
  - The traveling salesman problem and many of its variations are hard to solve.
- Heuristics, or approximate algorithms, are often used for problems that are hard to solve.

Outline

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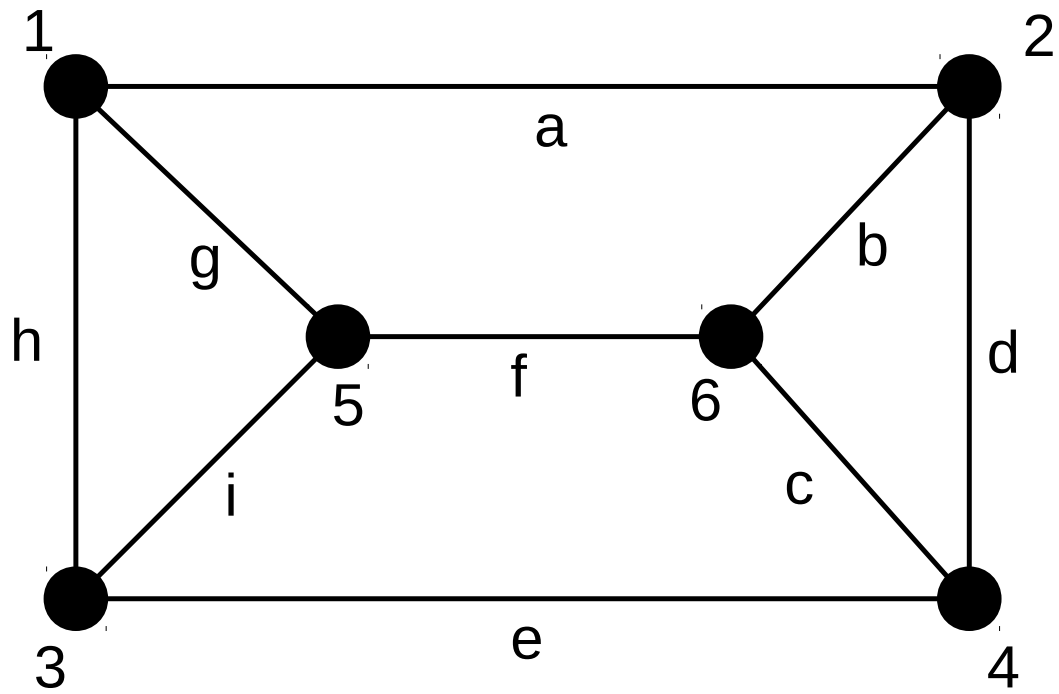
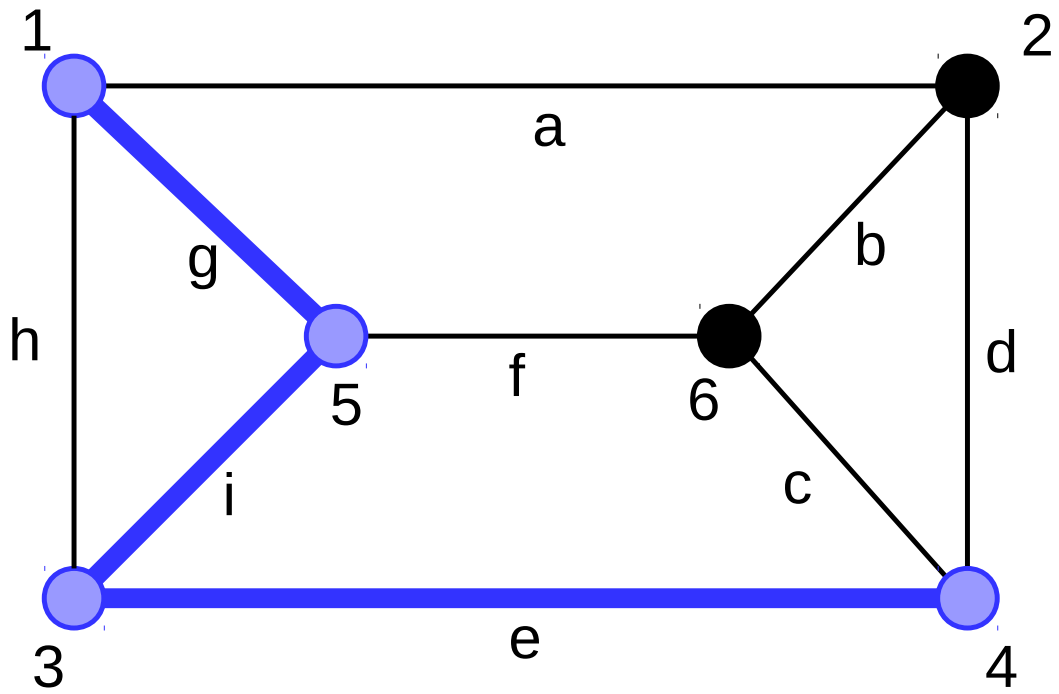
Graph Theory

- A *graph* is a pair of sets $G = (V, E)$.
- V is a set of vertices (sometimes called nodes).
- E is a set of edges connecting the vertices.
- Each edge has a cost value.



Graph Theory

- A *path* is a sequence of connected vertices and edges with no repeated vertices
- A *cycle* is a path except that the first vertex is the same as the last vertex



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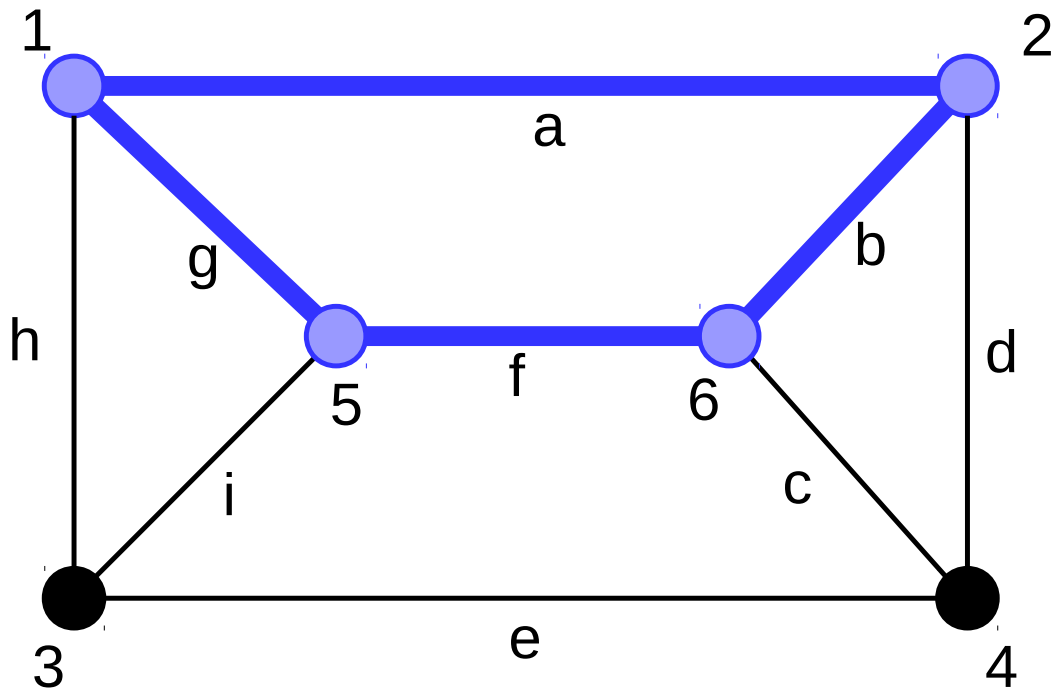


Computational Complexity

- A decision problem returns a ‘yes’ or ‘no’ answer.
- Polynomial time algorithms increase in steps by a factor of n^k as input size n increases.
- Decision problems that can be verified by a polynomial time algorithm are in the class NP.
- Verifying is not solving!
  - "Given a bag of 1000 keys and a door, is there a key in the bag that unlocks the door?"
  - "Given one key, does it unlock the door?"
- Problems that are NP-hard are at least as hard as every problem in NP.

The Generalized Traveling Salesman Problem

- The traveling salesman problem (TSP) is an NP-hard problem.
  - The goal is to find the minimum-cost cycle with all vertices.
- The generalized traveling salesman problem (GTSP) is a generalization of the TSP.
  - Vertices are divided into disjoint subsets.
  - The goal is to find the shortest cycle that contains one vertex from each subset.

Outline

Local-Global Search

- Pop and Iordache
- A sequence $(V_{k_1}, V_{k_2}, ..., V_{k_n})$ of vertex subsets is chosen.
- Subset V_{k_1} is duplicated and added to the end of the sequence.
- Shortest path that contains vertices from all subsets is found.
- Local-global search does not lead to an optimal solution.

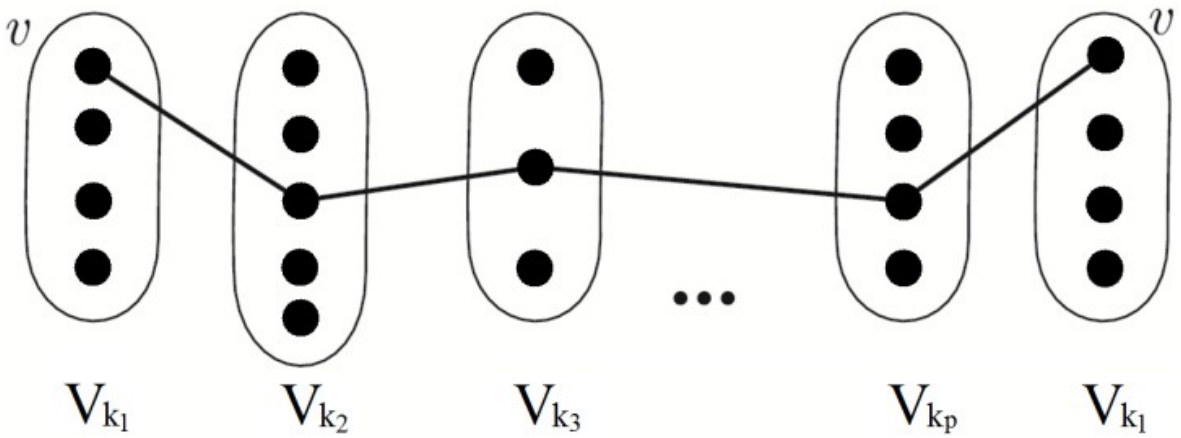
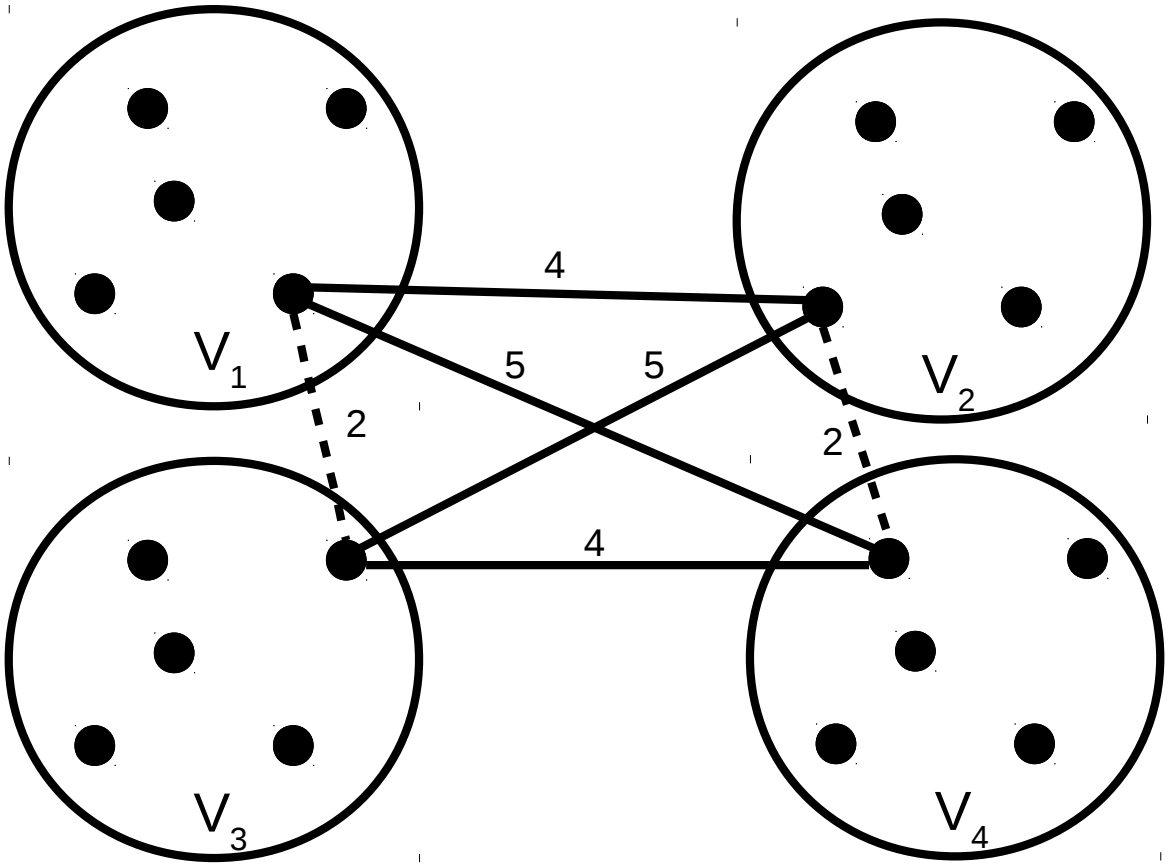


Figure: Adapted from Pop and Iordache

Local-Global Search

$$(V_1, V_2, V_3, V_4, V_1)$$

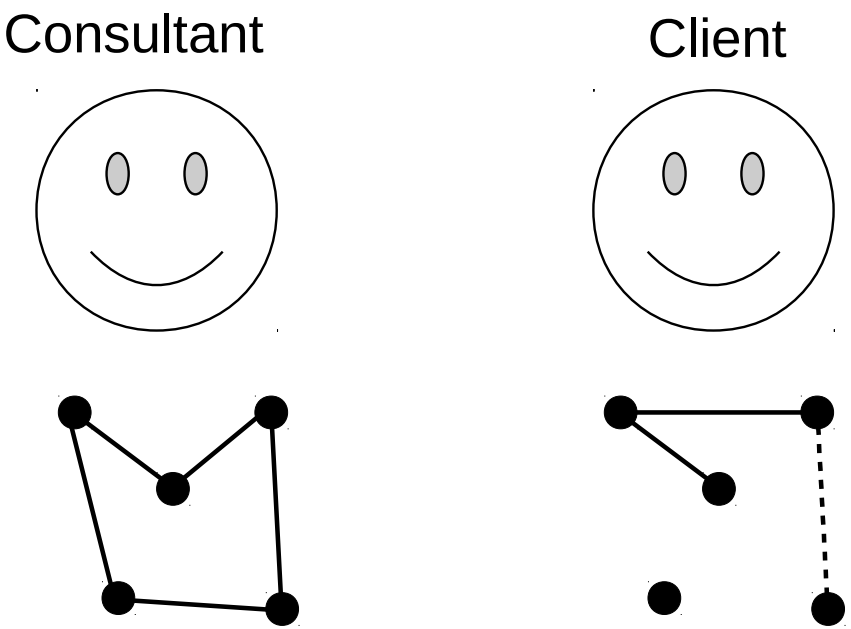


Consultant Guided Search

- Swarm Intelligence
- Simulated individuals each take on roles of “consultant” and “client.”
- Each virtual client chooses a consultant based on consultants’ “reputation.”
- The consultant gives the client suggestions.
- The client may or may not take the consultant’s suggestion.
- The consultant’s reputation changes based on the performance of the clients.

The Hybrid Algorithm

- The consultant constructs a cycle of vertex subsets.
- The client builds a sequence of vertex subsets.
- The consultant suggests the next vertex subset.
- After a sequence is found, local-global search is used to find the best cycle given the sequence.
- Variant: Each edge has a confidence value.



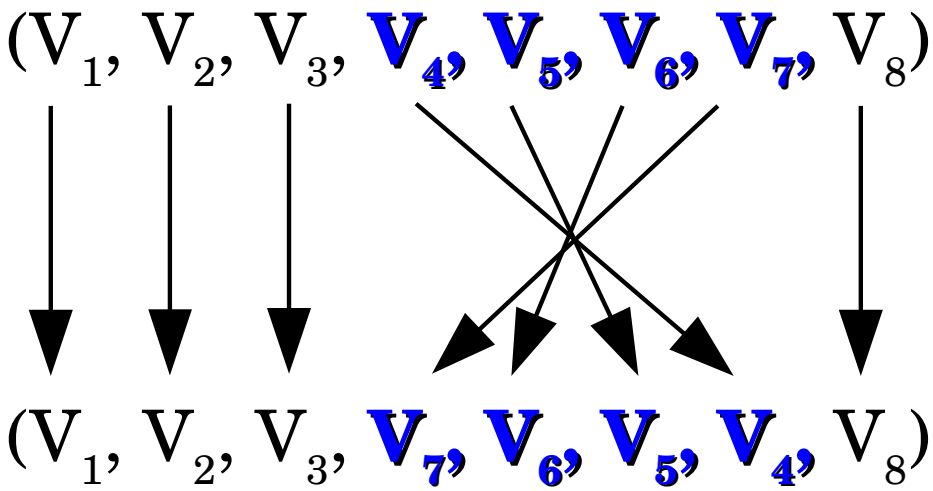
Results

- Regular algorithm and variant were compared to the best known algorithm at the time.
- Variant was statistically similar to the best known algorithm.
- Variant was much better in some cases.

Outline

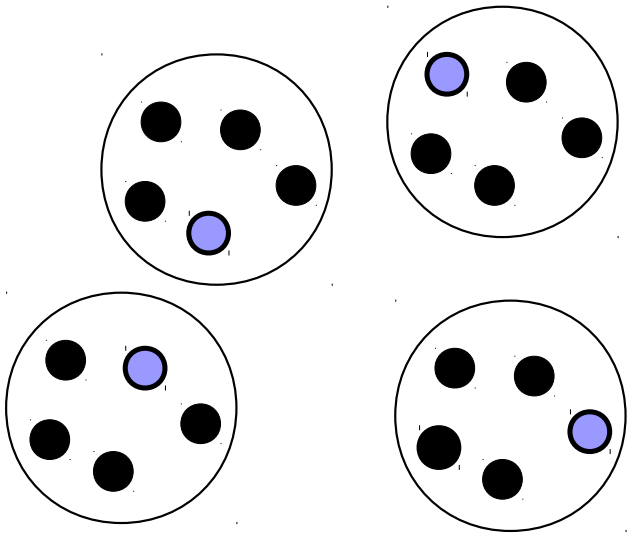
Cluster-Based Local Search

- CBLS starts with a sequence, finds best vertices within each vertex subset (or *cluster*).
- The sequence is changed after every iteration.
- The algorithm returns the best cycle after certain permutations are tried.



Node-Based Local Search

- Starts with a vertex chosen in each subset, then finds the best sequence of vertices.
- Exact or approximate algorithms can be used to find the sequence of vertices.
- The vertices chosen are changed after every iteration.
- The algorithm returns the best cycle and vertices after certain vertices are tried.



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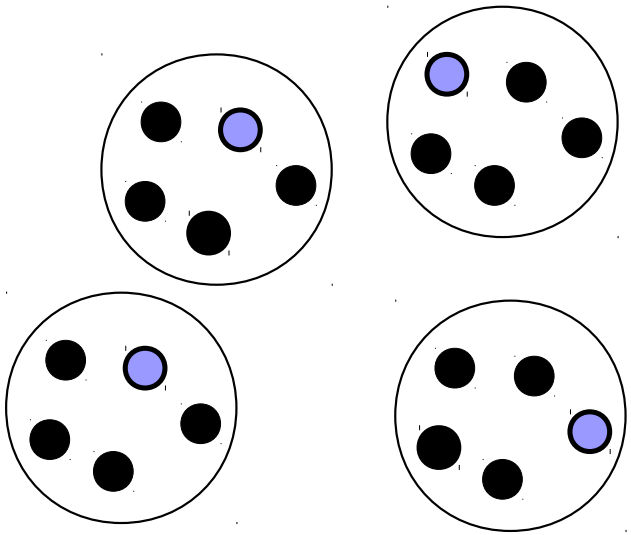


Variable Neighborhood Search

- VNS combines the two search algorithms.
- Cluster-based local search is used first.
- Once CBLS has found a local optimum, node-based local search is used.

Advantages and Disadvantages

- Cluster-based local search and node-based local search could each be used alone.
- Which method is best in what situations?
- Why use variable-neighborhood search?
- Pourhassan and Neumann, University of Adelaide, Australia

Advantages of Node-Based Local Search

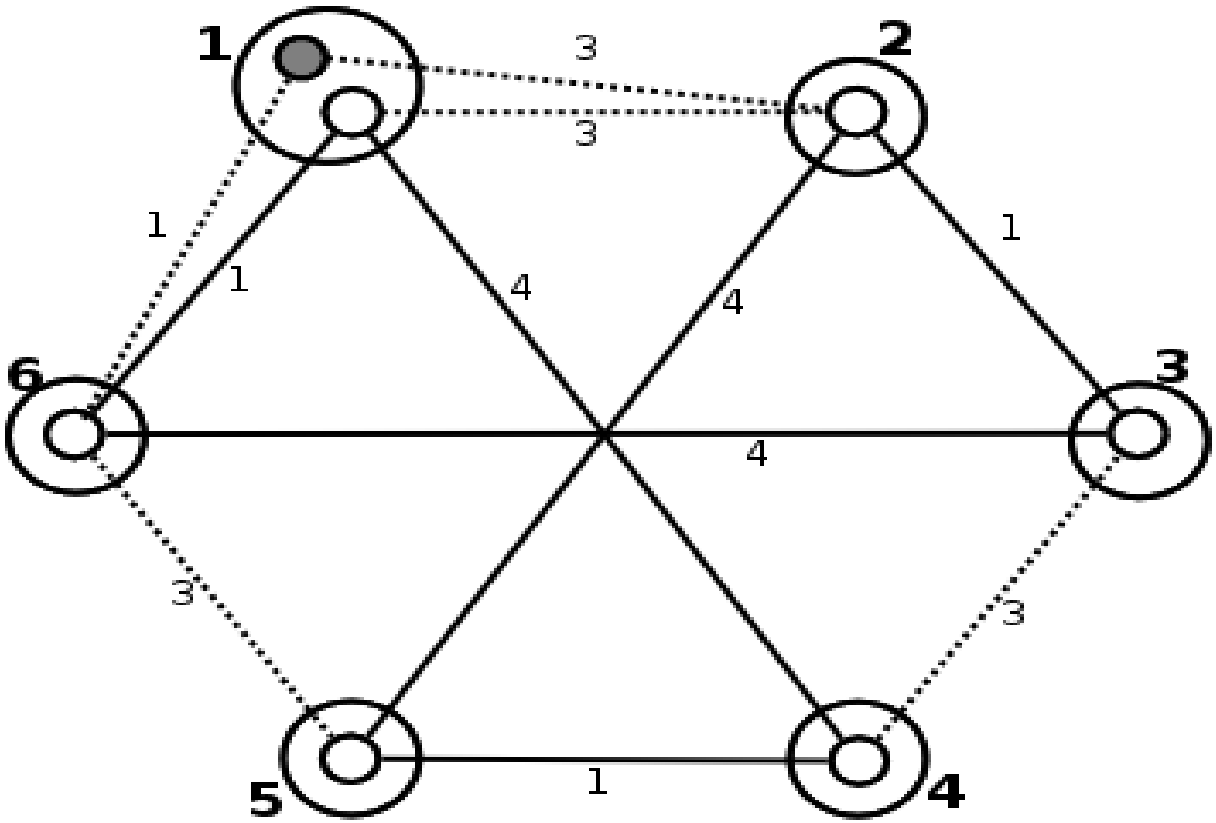
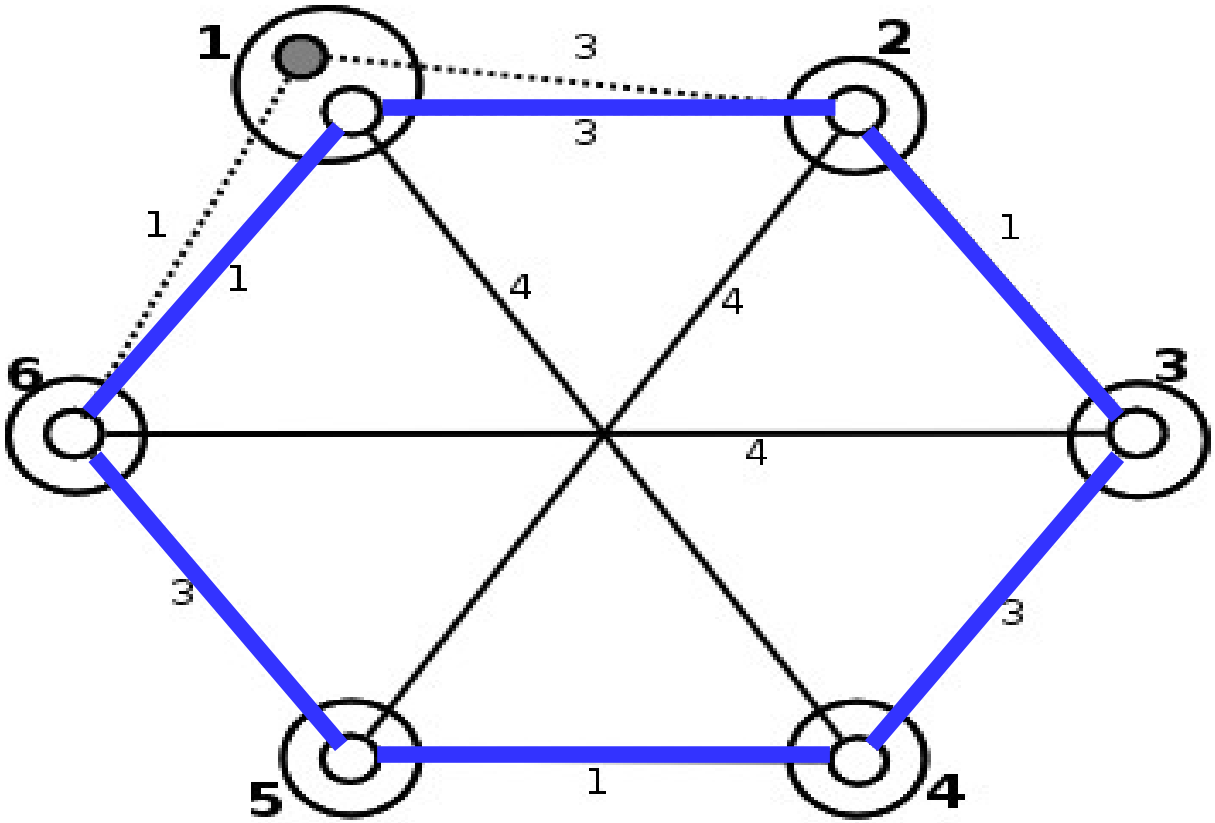


Figure: An instance that is hard for Cluster-Based Local Search (Taken from Pourhassan and Neumann)

Advantages of Node-Based Local Search



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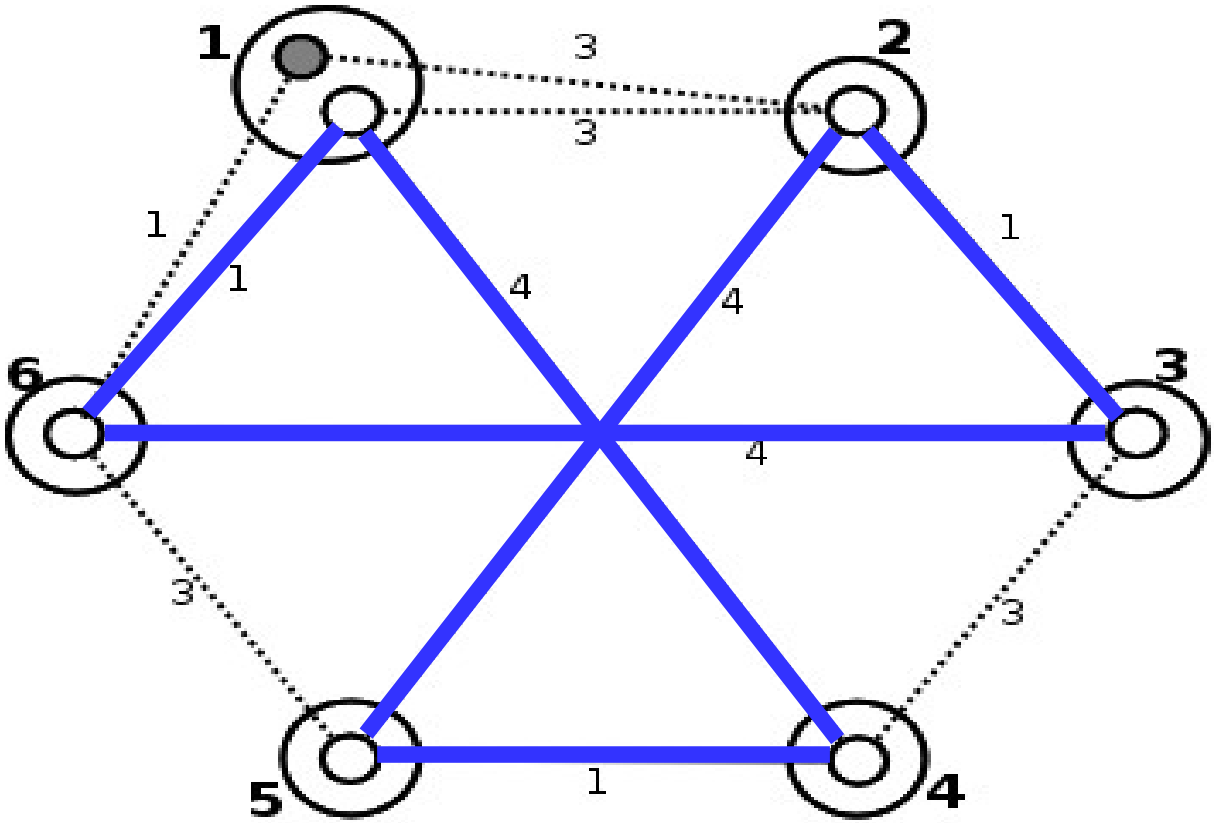

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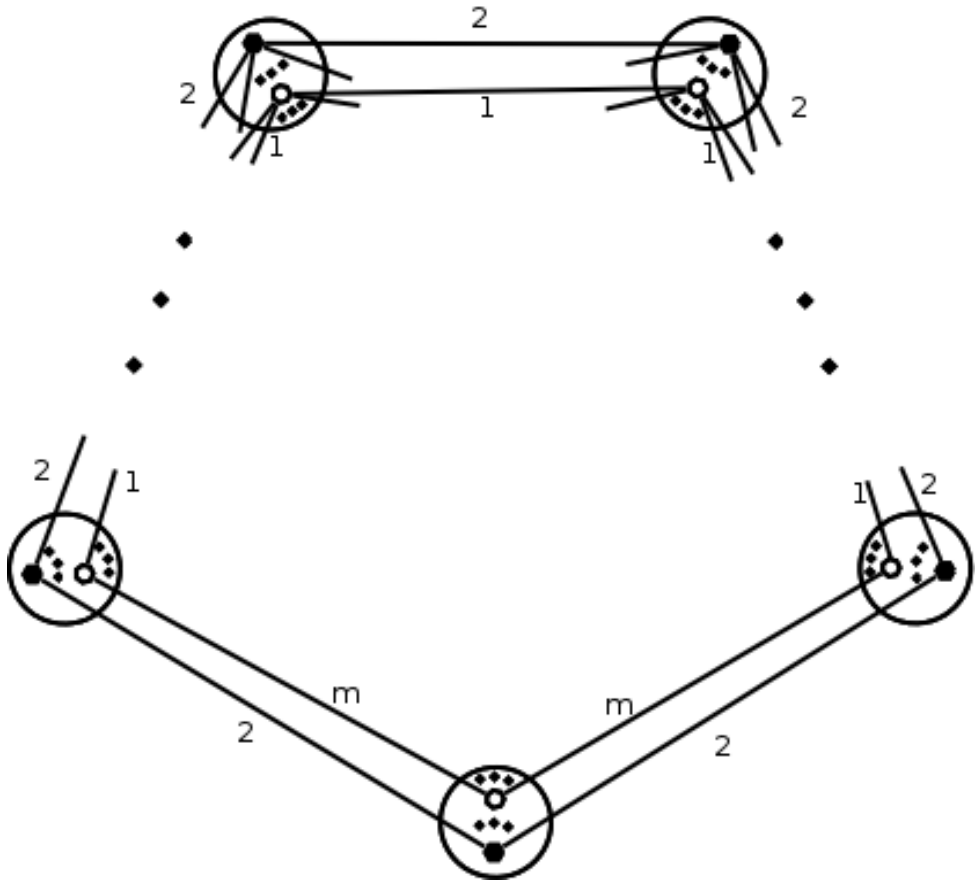

Figure: An instance that is hard for Node-Based Local Search, where m is the total number of subsets (Taken from Pourhassan and Neumann)

Advantages of Variable Neighborhood Search

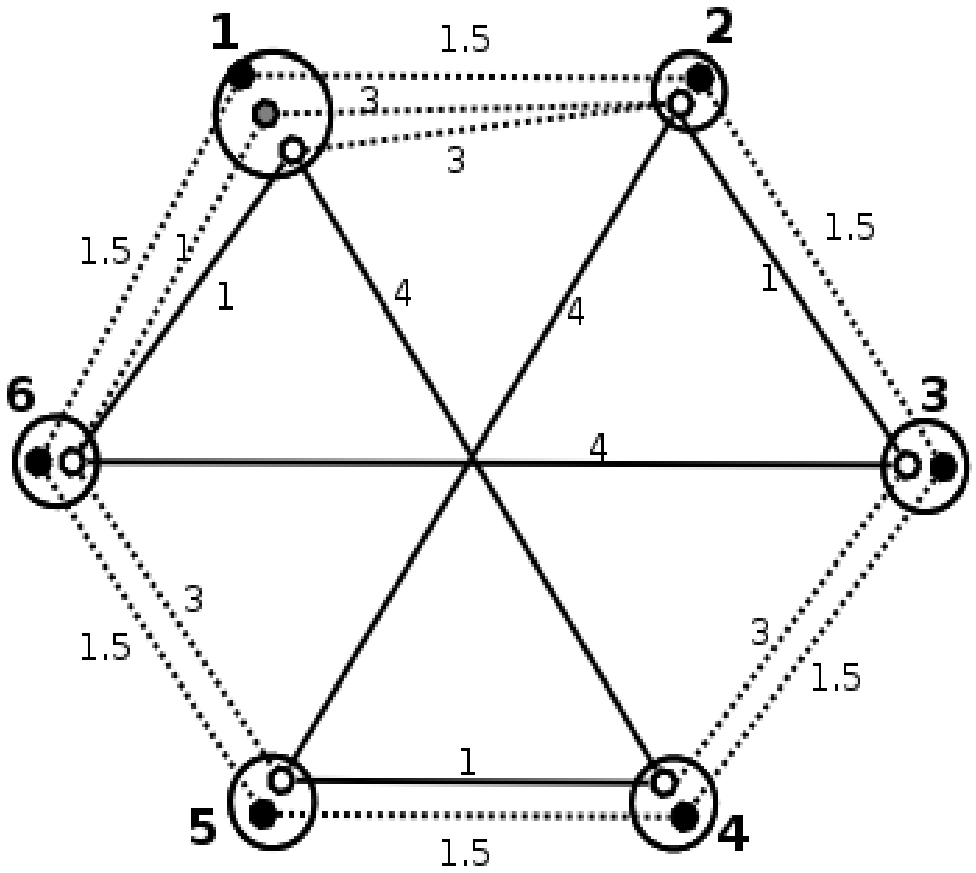

Figure: An instance that is hard for either but easy with VNS (taken from Pourhassan and Neumann)

Outline

Conclusions

- The GTSP is computationally intensive.
- Heuristics are effective for many practical purposes.
- Different heuristics have different strengths.
- Combining heuristics can increase effectiveness.

References


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Thank you!

Questions?