Rainbow Tables

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- Rainbow table
- Create rainbow tables (offline stage)
- Use rainbow tables (online stage)
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Introduction & Background

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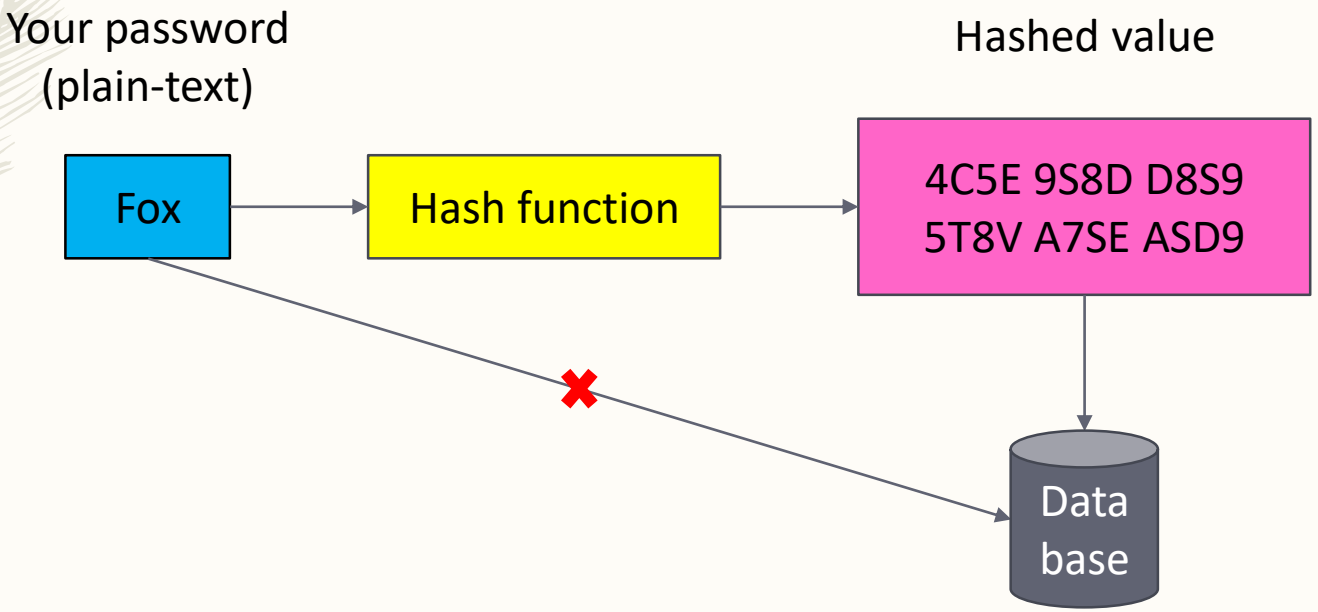
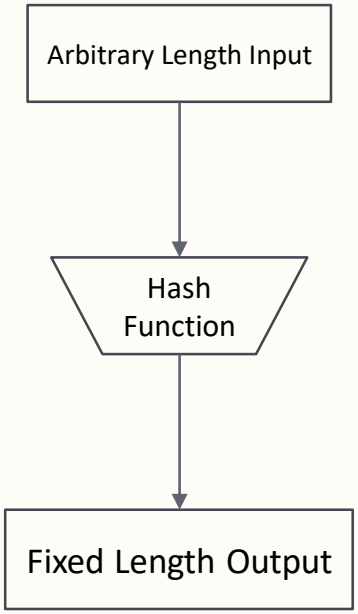


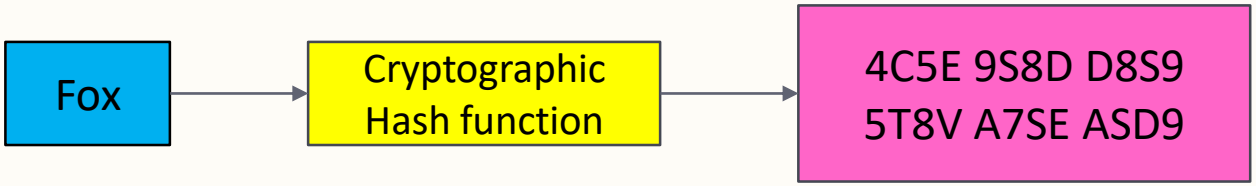
Introduction & Background

- Hash function
  - Map data of arbitrary size onto data of fixed size



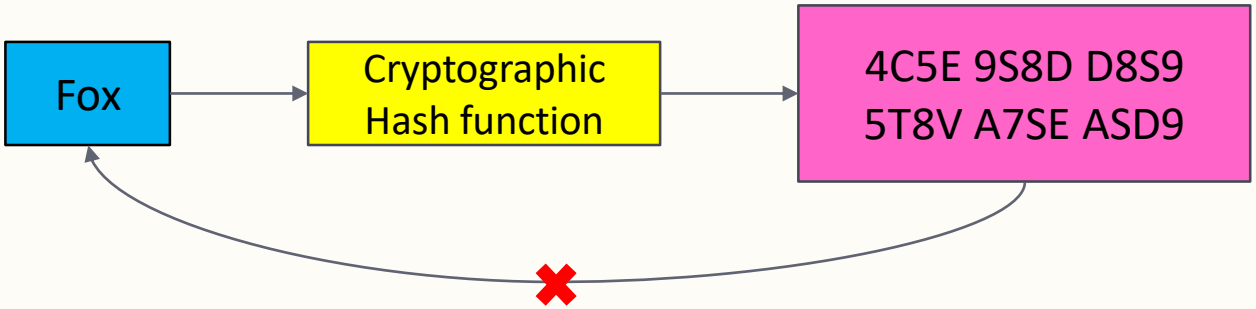
Introduction & Background

- Cryptographic hash function
  - Same plain-text result in same hashed value;



Introduction & Background

- Cryptographic hash function
  - Same plain-text result in same hashed value;
  - Fast to compute;
  - Infeasible to revert back to plain-text from hashed value;



Introduction & Background

- Cryptographic hash function
  - Same plain-text result in same hashed value;
  - Fast to compute;
  - Infeasible to revert back to plain-text from hashed value;
  - Small change(s) in plain-text will cause huge changes in hashed value;

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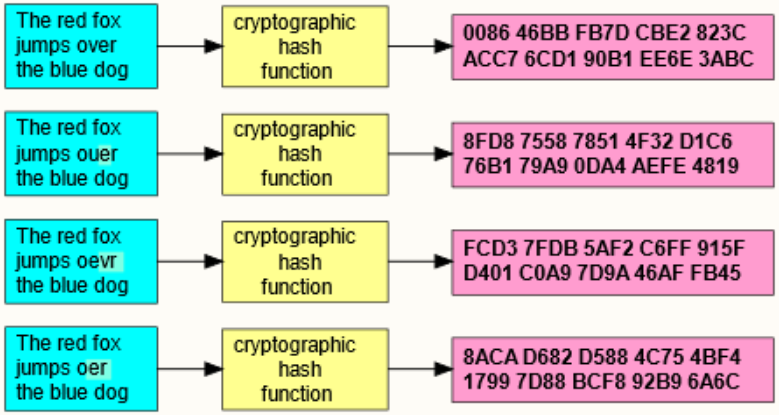
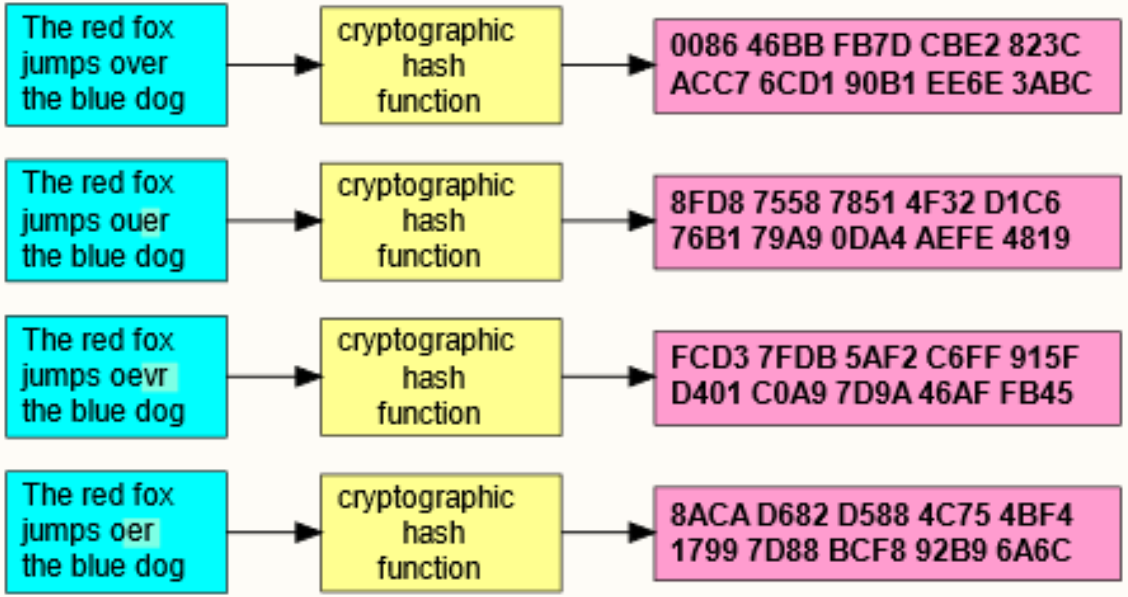


Introduction & Background

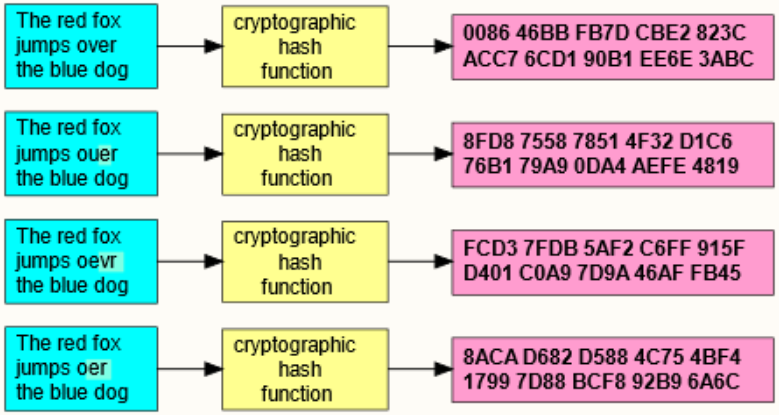


Introduction & Background

- Cryptographic hash function
  - Same plain-text result in same hashed value;
  - Fast to compute;
  - Infeasible to revert back to plain-text from hashed value;
  - Small change(s) in plain-text will cause huge changes in hashed value;
  - Infeasible to find two different plain-text with the same hashed value.

Introduction & Background

- Cryptographic hash function
  - Same plain-text result in same hashed value;
- Brute force



Introduction & Background

Password length	Alphanumeric	Days
1	62	0
2	3844	0
3	238328	0
4	14776336	0
5	916132832	0
6	56800235584	0
7	3.52 e+12	1
8	2.18 e+12	42
9	1.35 e+12	2599
10	8.39 e+12	161156

- Lookup table
- 8 digits; 62 characters
- 222 trillion combinations
- 2 quadrillion bytes
  = 1800 terabytes
- 32 days with i7

Plain-text	Hash value
aaaaaaaa	7D9SXF
aaaaaaab	7WS4G5
aaaaaaac	5F2V6D
…	…
00000000	8CVIDF
00000001	1QSD9F
…	…

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Rainbow table

- Based on the idea introduced by Martin Hellman in 1980
- Improved by Philippe Oechslin in 2003
- Two stages
  - Offline stage
  - Online stage

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Rainbow table

- Offline stage
  - Create tables
    - *Chains*
    - *Cryptographic hash function (h)*
    - *Reduction function (r)*
    - *Collision*

Rainbow table

- Chains
  - Only start point t_0 and end point h_2 matters
  - Tables look like:

$$S_0 = t_0 \xrightarrow{h} h_0 \xrightarrow{r_0} t_1 \xrightarrow{h} h_1 \xrightarrow{r_1} t_2 \xrightarrow{h} h_2 = E_0$$

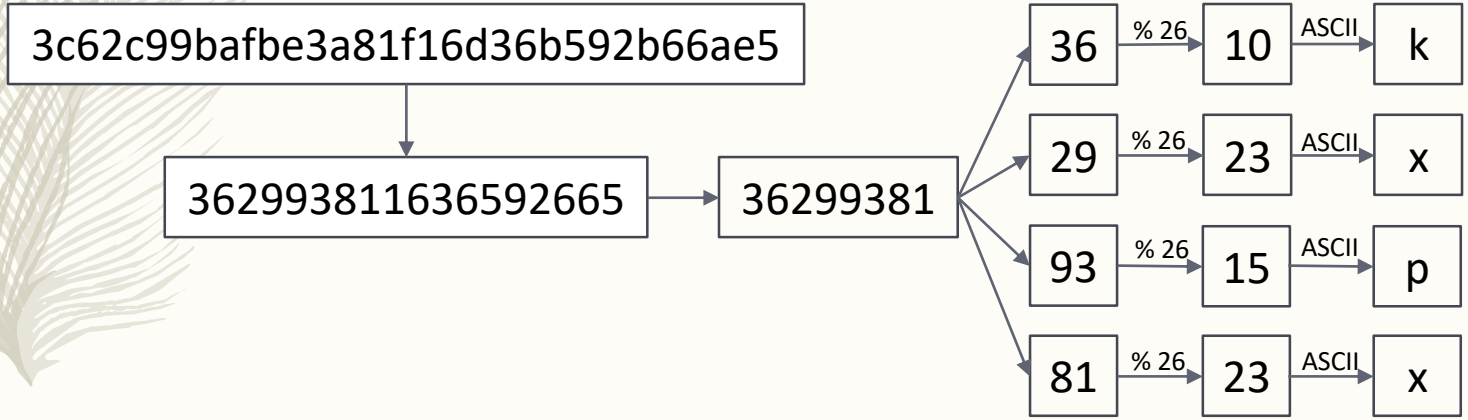
Text		Hash
aaaa	⇝	3c62c99bafbe3a81f16d36b592b66ae5
aaab	⇝	7991ef7c61d85c505adb63b70553b0ce
aaac	⇝	ef834a79dc0ac78e355859ec4f698d0b
aaad	⇝	3bb75d8a8effe7c5e18aC6670a3c3346
⋮	⇝	⋮

Rainbow table

- Cryptographic hash function (h)
  - MD5, SHA-1, SHA-2, SHA-3, BLAKE2, and etc.
  - Slight different in time due to different algorithm used

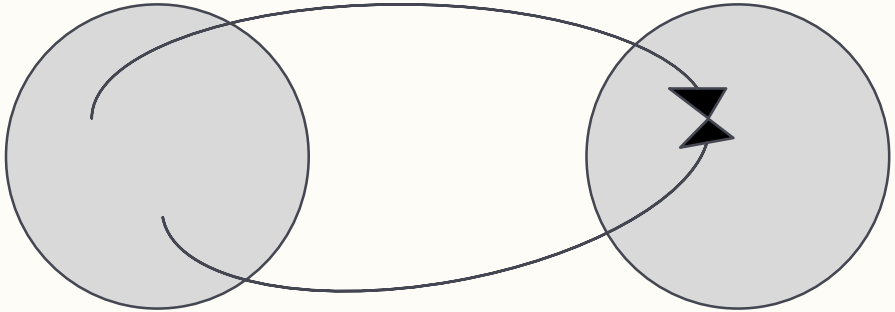
Rainbow table

- Reduction function (r)
  - Map data from the set of hashed values to the set of all plain-texts
  - Example:



Rainbow table

- Collision
  - When we map two different thing into the same value, no matter which way, it is called collision

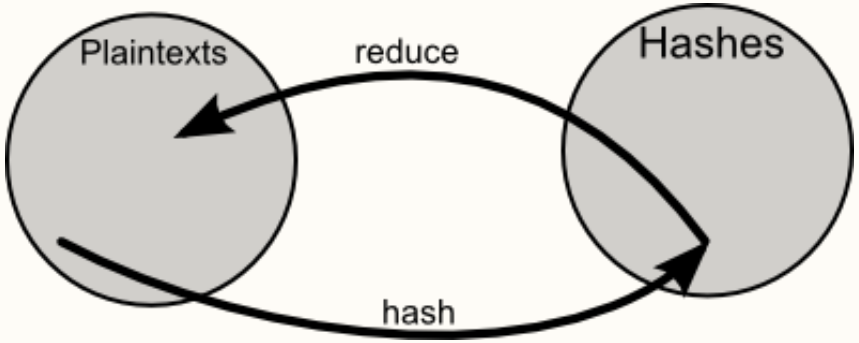


Rainbow table

- Collision
  - The size of the set of plain-texts and the set of hashed value will be different normally
  - When we try to map from a larger set onto a smaller set, collisions occur more frequently (Pigeonhole principle)

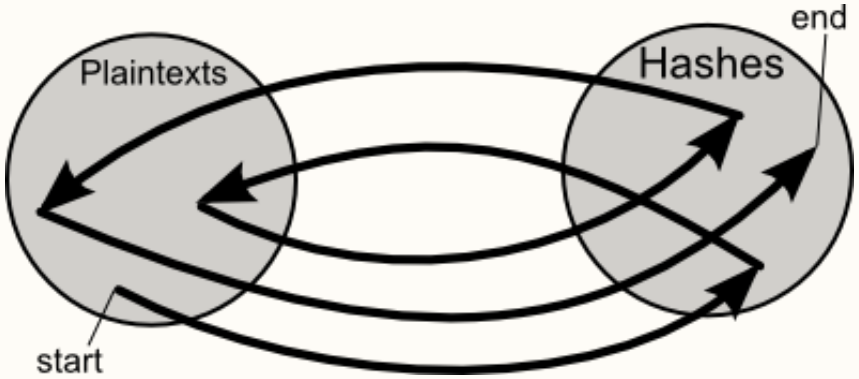
Rainbow table

– Generate steps:



Rainbow table

- Generate steps:



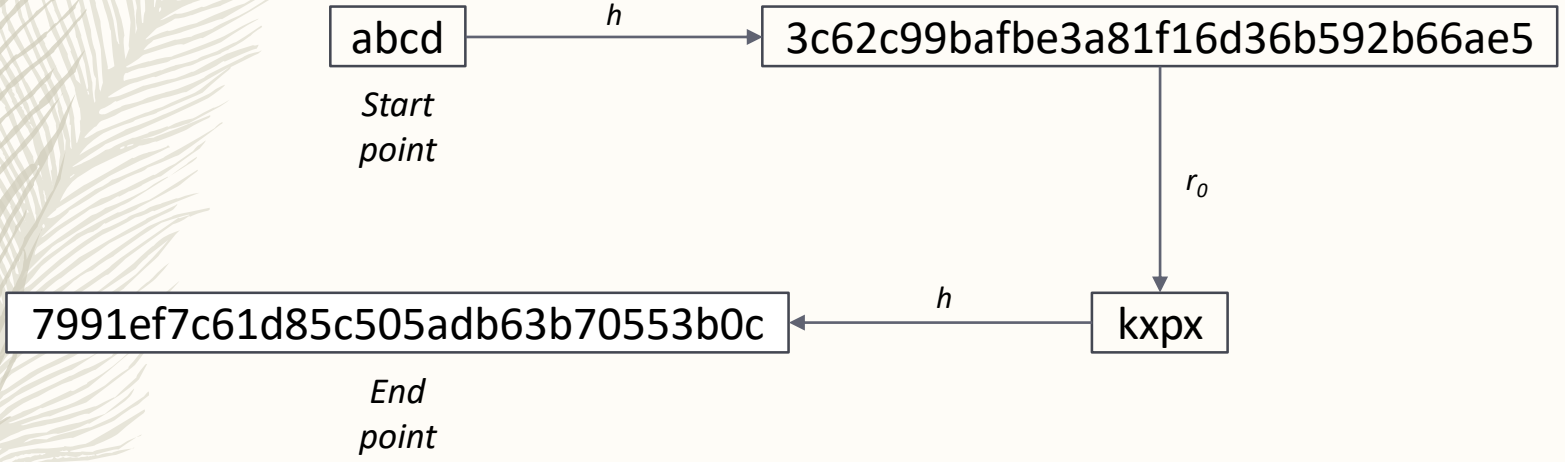
Rainbow table

- The structure of a rainbow table:

$$
\begin{array}{|c|c|c|c|c|c|c|c|c|c|}
\hline
S_0 = t_{0,0} & \xrightarrow{h} & h_{0,0} & \xrightarrow{r_0 \circ h} & h_{0,1} & \xrightarrow{r_1 \circ h} & \dots & \xrightarrow{r_{n-2} \circ h} & h_{0,n-1} & \xrightarrow{r_{n-1} \circ h} & h_{0,n} = E_0 \\
S_1 = t_{1,0} & \xrightarrow{h} & h_{1,0} & \xrightarrow{r_0 \circ h} & h_{1,1} & \xrightarrow{r_1 \circ h} & \dots & \xrightarrow{r_{n-2} \circ h} & h_{1,n-1} & \xrightarrow{r_{n-1} \circ h} & h_{1,n} = E_1 \\
\vdots & & \vdots & & \vdots & & \vdots & & \vdots & & \vdots \\
S_j = t_{j,0} & \xrightarrow{h} & h_{j,0} & \xrightarrow{r_0 \circ h} & h_{j,1} & \xrightarrow{r_1 \circ h} & \dots & \xrightarrow{r_{n-2} \circ h} & h_{j,n-1} & \xrightarrow{r_{n-1} \circ h} & h_{j,n} = E_j \\
\vdots & & \vdots & & \vdots & & \vdots & & \vdots & & \vdots \\
S_m = t_{m,0} & \xrightarrow{h} & h_{m,0} & \xrightarrow{r_0 \circ h} & h_{m,1} & \xrightarrow{r_1 \circ h} & \dots & \xrightarrow{r_{n-2} \circ h} & h_{m,n-1} & \xrightarrow{r_{n-1} \circ h} & h_{m,n} = E_m \\
\hline
\end{array}
$$

Rainbow table

– Example:



Rainbow table

- Example (collision):

  t_0 = *"abcd"* → $r_0(h(t_0))$ = *"defg"*

  t_1 = *"abdc"* → $r_0(h(t_1))$ = *"defg"*

  we will have two chains with different start points but the same end point.

  - Clean table
    - Only keep one chain with the same end point.

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Rainbow table

- Online stage
  - Actual search
  - Example:

    We have a rainbow table with chain length = 3:

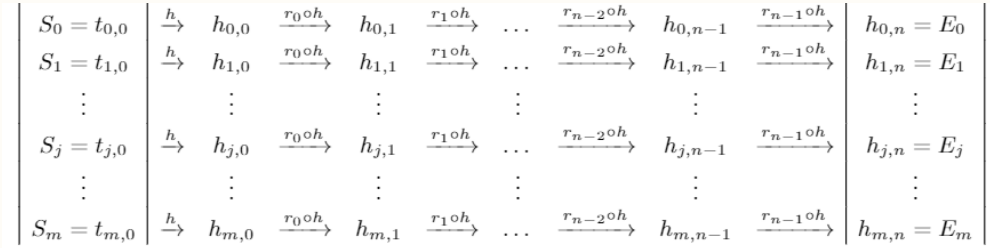
    Rainbow table: $t_0 \sim \rightarrow h_2$

    Chain: $t_0 \overset{h}{\rightarrow} h_0 \overset{r_0}{\rightarrow} t_1 \overset{h}{\rightarrow} h_1 \overset{r_1}{\rightarrow} t_2 \overset{h}{\rightarrow} h_2$

    Assuming the password we want to search for hash value:

    3C62C9 & 77CC7F

Text	Hash
aaaa	3C62C9
aaab	7991EF
aaac	EF834A
spkn	3BB75D

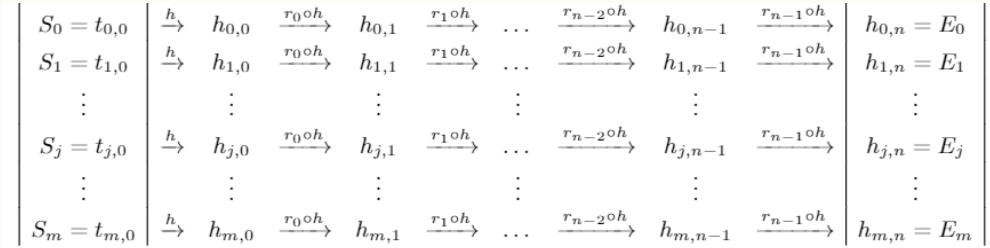


Chain: $t_0 \xrightarrow{h} h_0 \xrightarrow{r_0} t_1 \xrightarrow{h} h_1 \xrightarrow{r_1} t_2 \xrightarrow{h} h_2$

Text	Hash
aaaa	3C62C9
aaab	7991EF
aaac	EF834A
spkn	3BB75D

1. According to the given hash value y=h(x)= 3C62C9, search through the column for the ending points in all tables;
2. We find it at the first row with the plain-text “aaaa”;
3. We create a chain from “aaaa”, and search for the original hash (3C62C9) in the chain; if we find it, it returns the password at a given index (indexHash -1). The chain is:

   $aaaa \xrightarrow{h}$ 584C19 $\xrightarrow{r_0}$ kcyl $\xrightarrow{h}$ 48950E $\xrightarrow{r_1}$ **puwr** $\xrightarrow{h}$ 3C62C9
4. In this case, the plain-text we wanted is “puwr”.



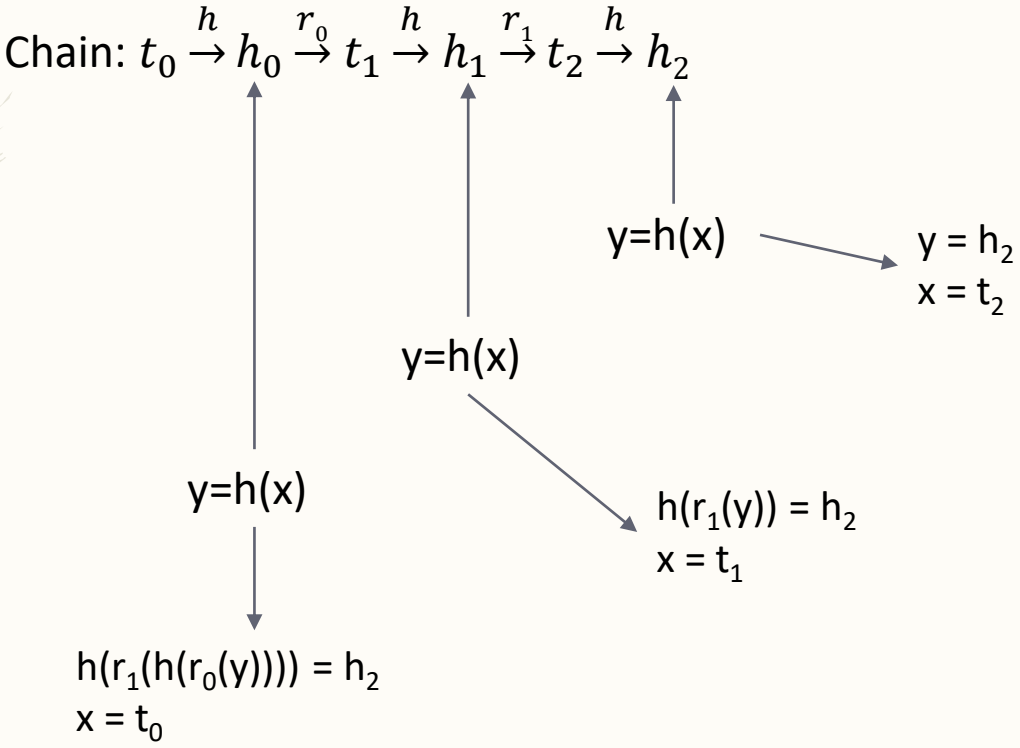
Chain: $t_0 \xrightarrow{h} h_0 \xrightarrow{r_0} t_1 \xrightarrow{h} h_1 \xrightarrow{r_1} t_2 \xrightarrow{h} h_2$

Text	Hash
aaaa	3C62C9
aaab	7991EF
aaac	EF834A
spkn	3BB75D

1. According to the given hash value y=h(x)= 77CC7F, search through the column for the ending points in all tables;
2. There is no matched E_j.
3. Compute the $h(r_1(77CC7F))$ = 48950E;
4. 48950E is not in the table;
5. Compute the $h(r_1(h(r_0(77CC7F))$ = 3BB75D;
6. We have a matched E_j, which is the last row with plain-text “spkn”;
7. We create a chain from “spkn”, and search for the original hash (77CC7F) in the chain; if we find it, it returns the password at a given index (indexHash -1). The chain is:

   spkn $\xrightarrow{h}$ 77CC7F $\xrightarrow{r_0}$ puwr $\xrightarrow{h}$ 584C19 $\xrightarrow{r_1}$ uhtc $\xrightarrow{h}$ 3BB75D
8. In this case, the plain-text we wanted is “spkn”.

The location of y=h(x)



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Tests

- Chain length;

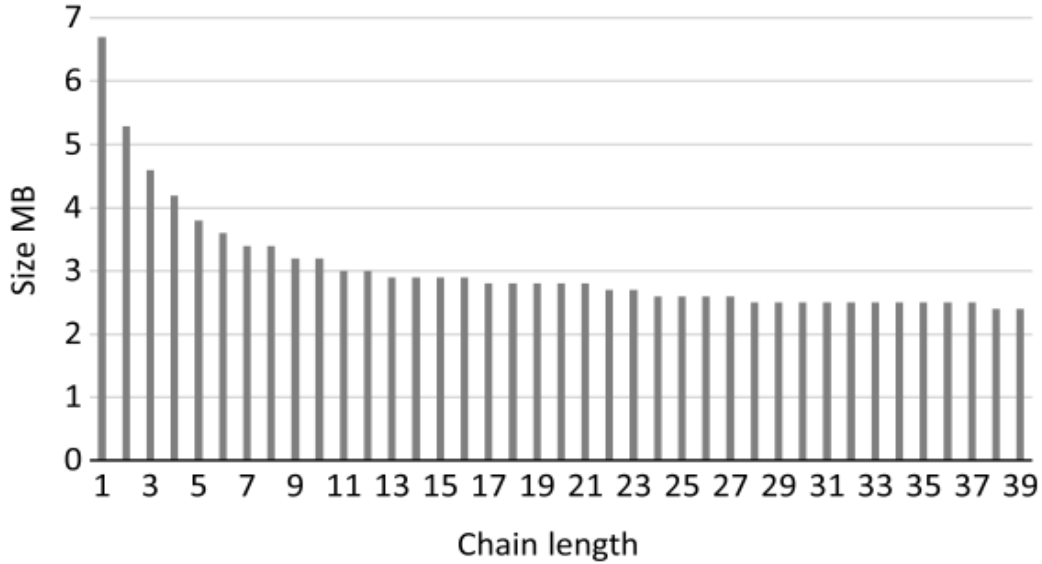

Fig. 2. The size of Rainbow Tables in dependence on the length of chain.

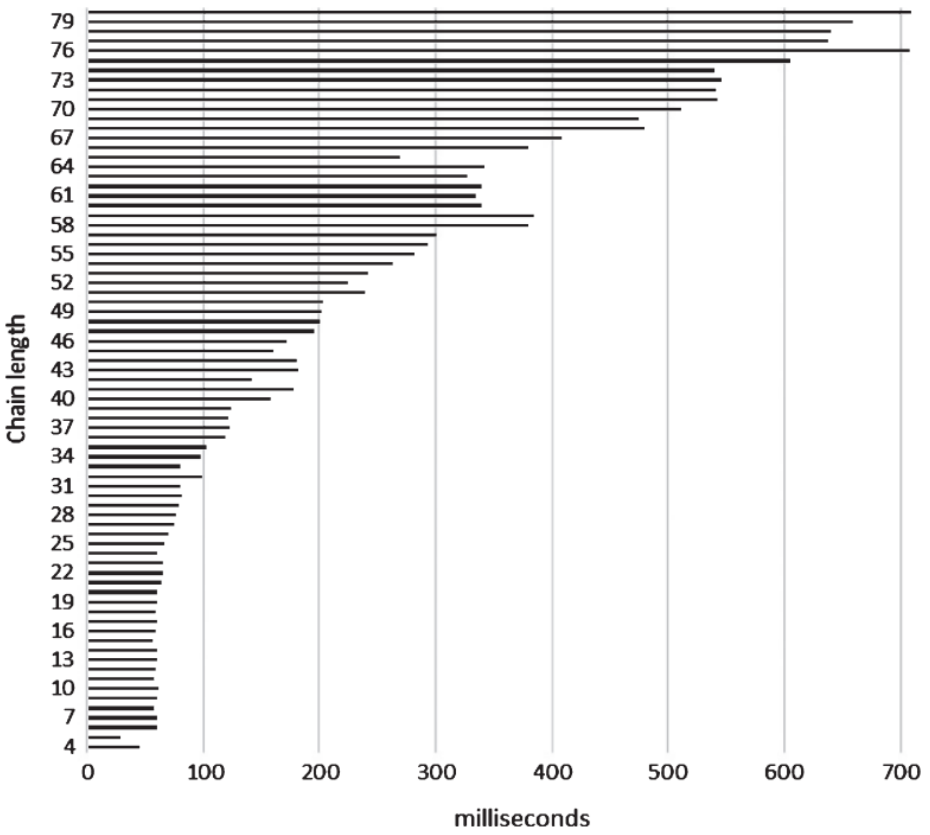

Fig. 1. Dependence on the chain length of the average time to find the password.

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Conclusion

- Time & space;

Acknowledgement

- Great thanks to Peter Dolan, KK Lamberty, and Elena Machkasova.

Reference

1. Horáleka, J., Holík, F., Horák, O., Petr, L., & Sobeslav, V. (2017). Analysis of the use of Rainbow Tables to break hash. Journal of Intelligent & Fuzzy Systems,32(2):1523 – 1534, 2017
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3. Cryptographic hash function. Cryptographic hashfunction — Wikipedia, the free encyclopedia, 2019.[Online; accessed 15-March-2019]

Q&A