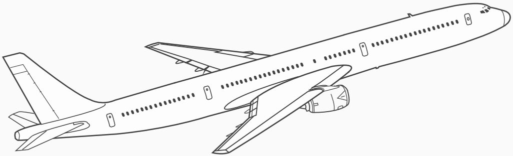


Scheduling Aircraft Departures to Avoid Enroute Congestion

Johannes Martinez

November 13, 2021

Image: NASA

Outline

- Background
  - Flight plans, air traffic control, sectors, hotspots
  - Linear programming, integer programming
- Big-M Constraint Formulation
- Graph Constraint Formulation
- Results

Flight Plans

- Airlines file plans for most flights.
- Plans include route waypoints and departure times.

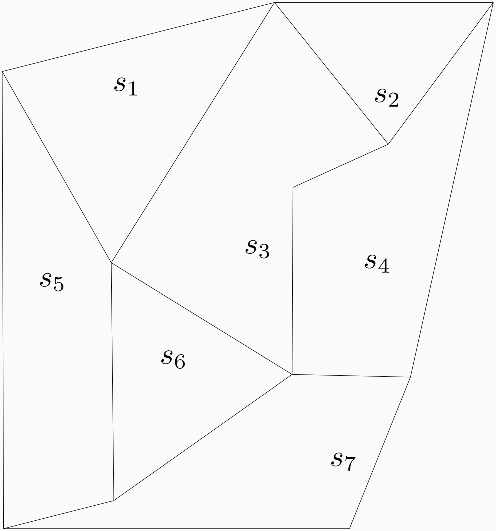
Image: Fabrizio Gandolfo (Wikimedia)

Enroute Air Traffic Control

- In the US, Air Route Traffic Control Centers are responsible for many flights at cruise altitude.
- Each center controls large swaths of airspace.

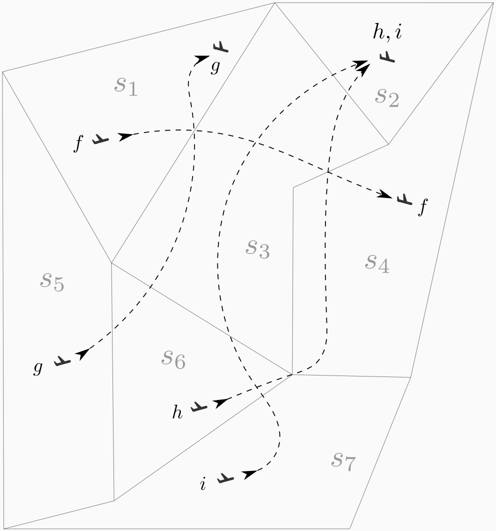
Image: FAA (ZDC ARTCC)

Areas and Sectors



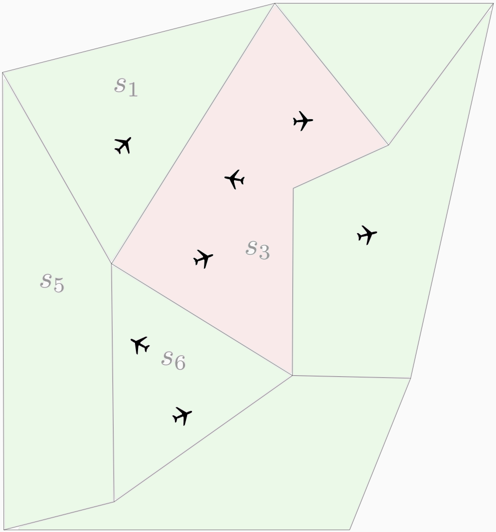
- Areas and sectors help air traffic control divide workload.
- We will consider a single layer of sectors.

Sectors and Flights



- Flight plans record the route and time of departure a flight wants.
- We model the relationship between sectors and flights.

Hotspots



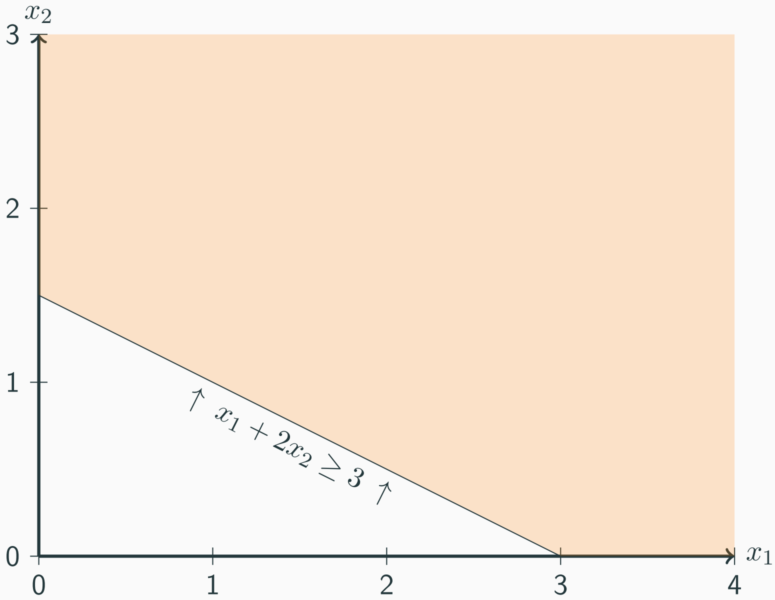
- We assume every sector can hold at most two aircraft.
- If this capacity is exceeded, we have a hotspot.

Linear Programming

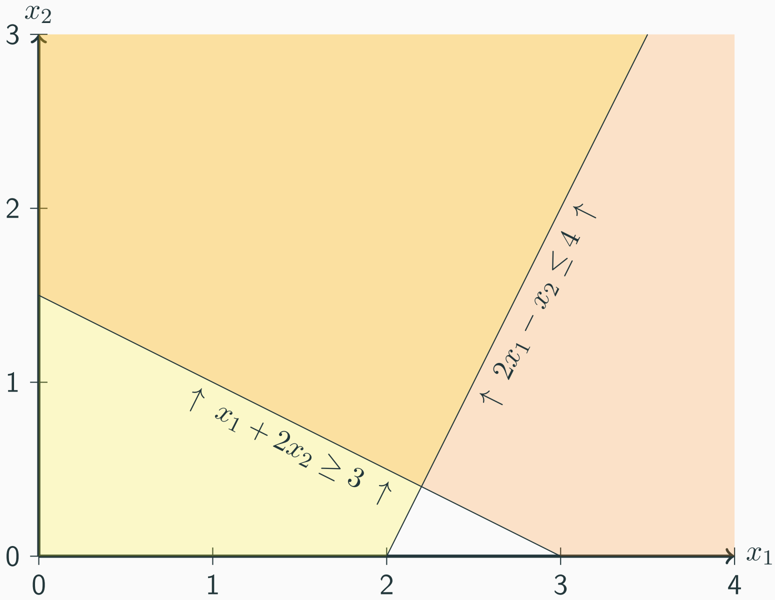
Linear Programming with Two Decision Variables

$$
\begin{array}{lll}
\text{Minimize} & x_1 + x_2 & \longrightarrow \text{Objective function} \\
\text{subject to} & -x_1 + 2x_2 \leq 2 & \longrightarrow \text{Constraint 1} \\
 & x_1 + 2x_2 \quad \geq 3 & \longrightarrow \text{Constraint 2} \\
 & 2x_1 - x_2 \quad \leq 4 & \longrightarrow \text{Constraint 3}
\end{array}
$$

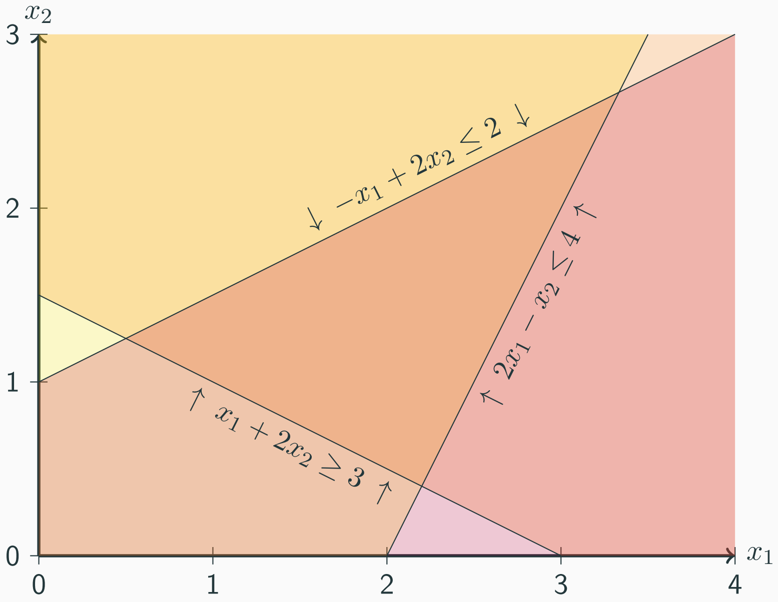
Linear Programming



Linear Programming

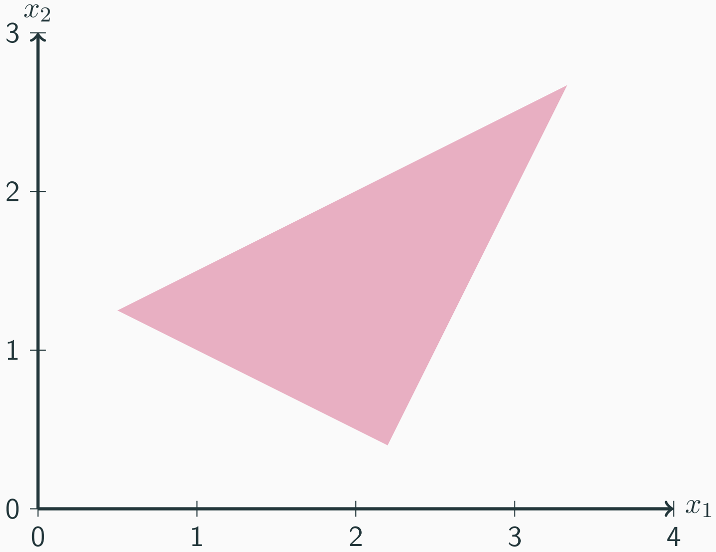


Linear Programming



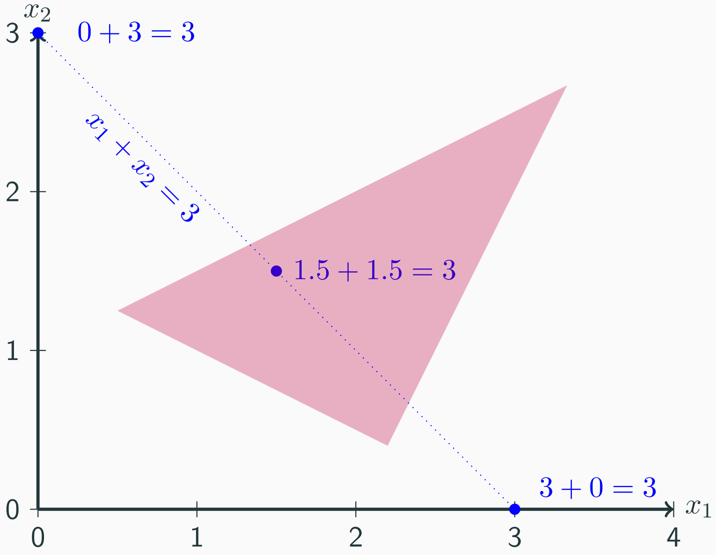
Linear Programming

Minimize $x_1 + x_2$



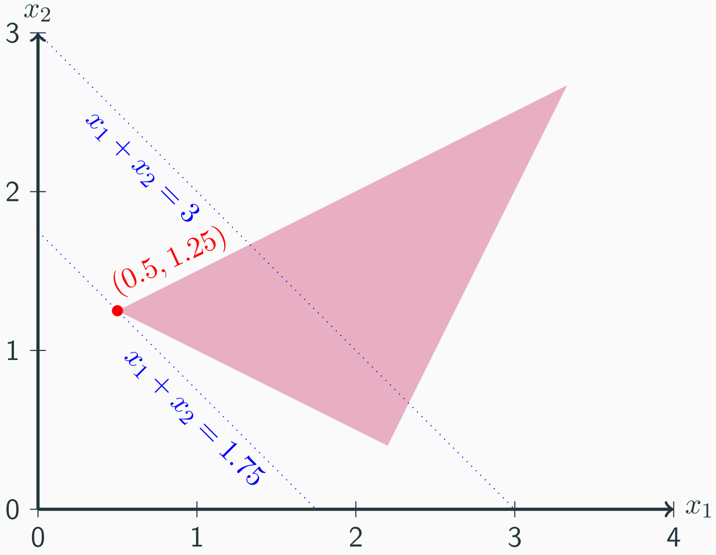
Linear Programming

Minimize $x_1 + x_2$



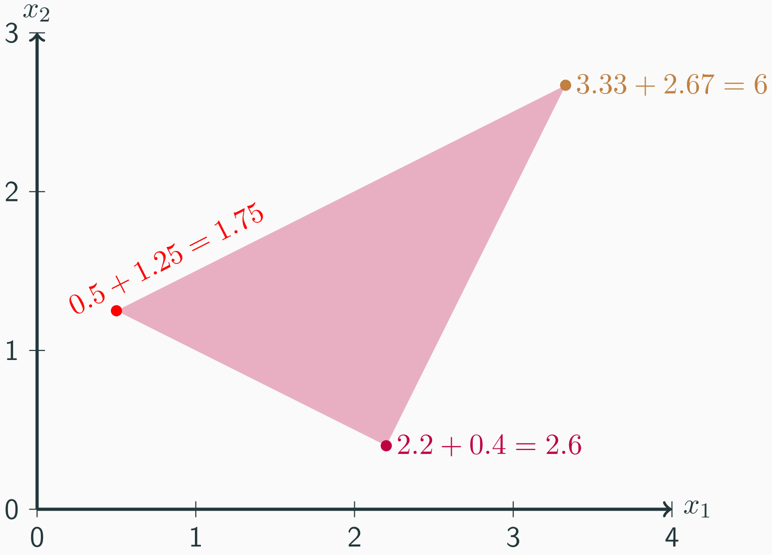
Linear Programming

Minimize $x_1 + x_2$



Linear Programming

Minimize $x_1 + x_2$

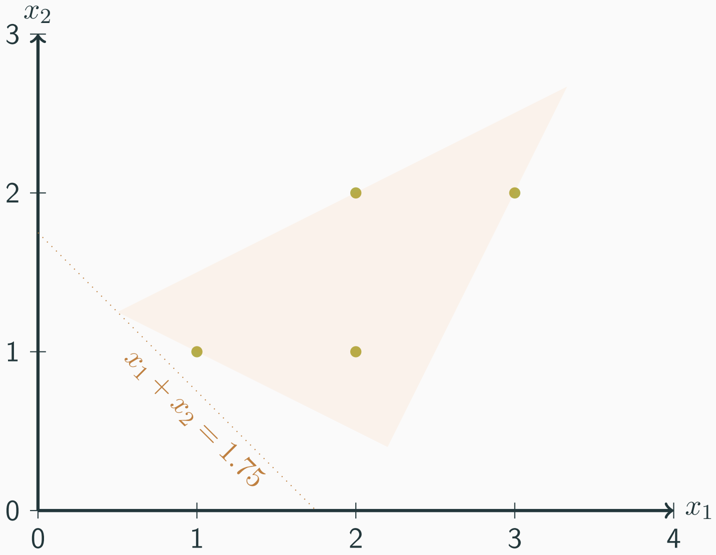


Scheduling Problems Require Discrete Variables

- Distinctions such as *concurrent* and *not concurrent* are binary.
- The feasible region for a discrete formulation cannot be continuous.

Integer Programming

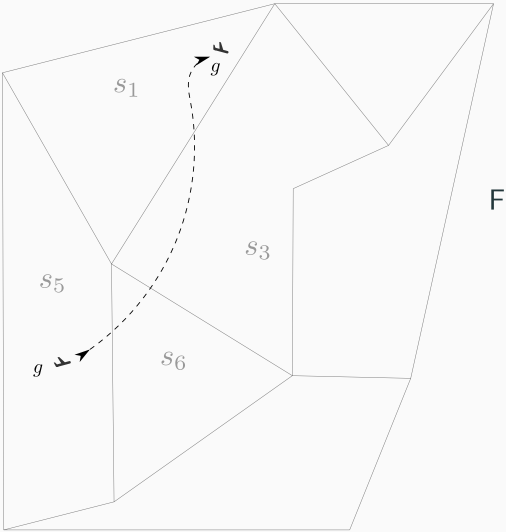
Minimize $x_1 + x_2$, where x_1 and x_2 are integers



Constraint Formulation for Flight Schedules

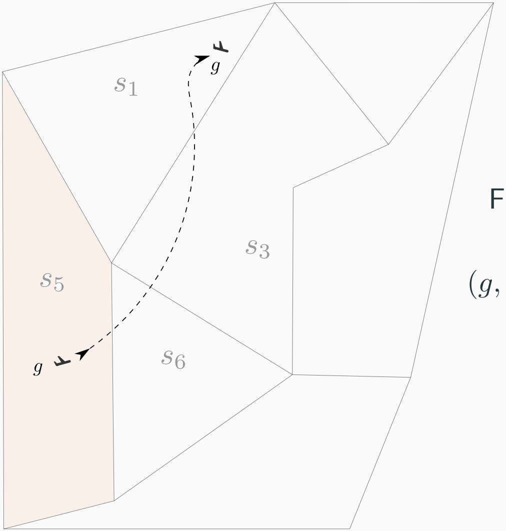
Mannino and Sartor, 2018

Route Nodes



Flight g, a route node sequence:

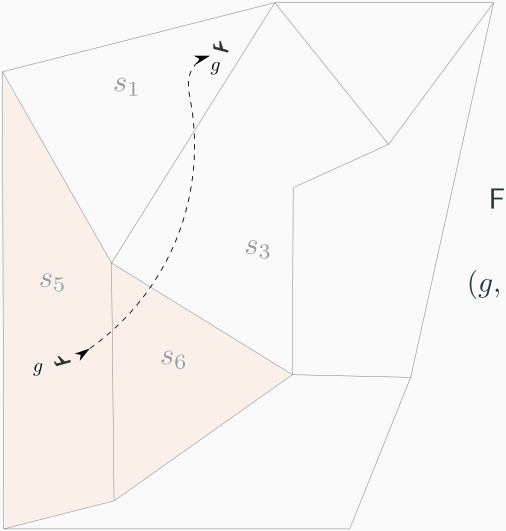
Route Nodes



Flight g, a route node sequence:

$(g,\ s_5),$

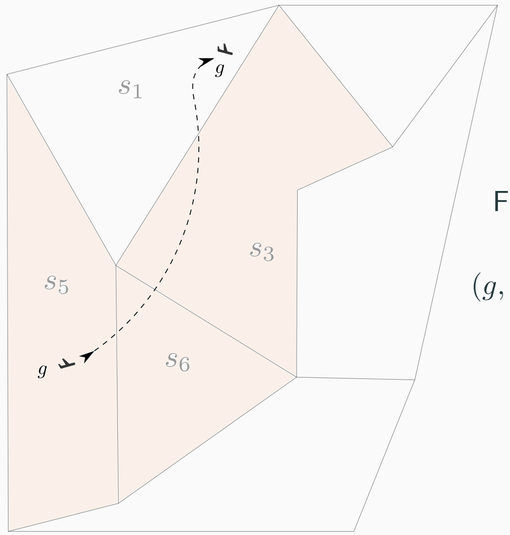
Route Nodes



Flight g, a route node sequence:

$(g,\ s_5),\ \ (g,\ s_6),$

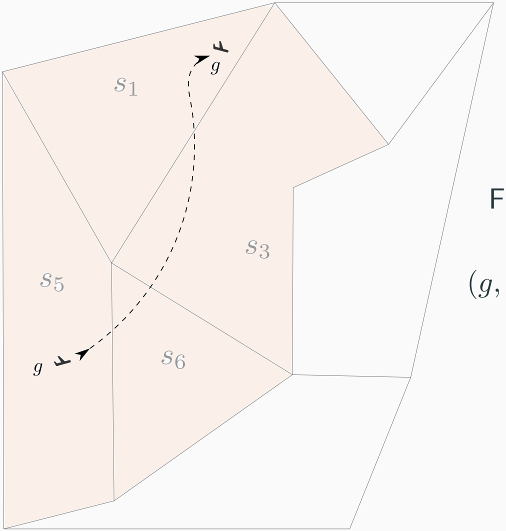
Route Nodes



Flight g, a route node sequence:

$(g,\ s_5),\ \ (g,\ s_6),\ \ (g,\ s_3),$

Route Nodes

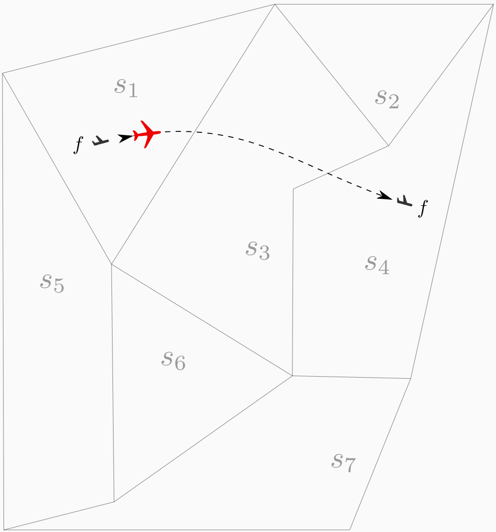


Flight g, a route node sequence:

$$(g,\ s_5),\ (g,\ s_6),\ (g,\ s_3),\ (g,\ s_1)$$

Schedules

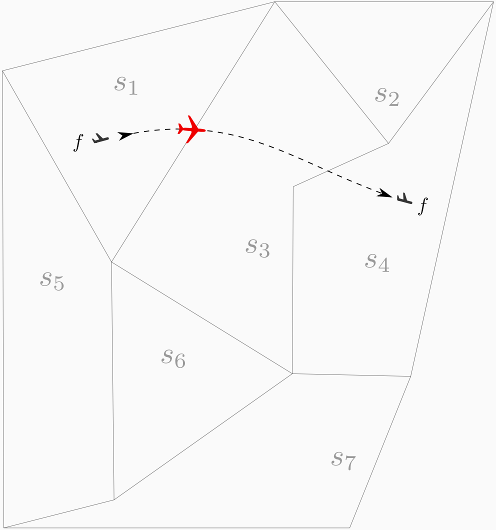
Schedule Nodes



Entry times for flight f:

$$t_{(f,\, s_1)} = 20$$

Schedule Nodes

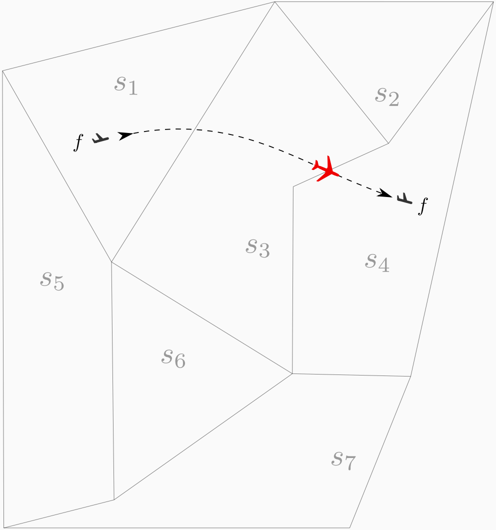


Entry times for flight f:

$$t_{(f,\, s_1)} = 20$$

$$t_{(f,\, s_3)} = 30$$

Schedule Nodes



Entry times for flight f:

$$t_{(f,\, s_1)} = 20$$

$$t_{(f,\, s_3)} = 30$$

$$t_{(f,\, s_4)} = 45$$

Constraints

Flights Cannot Depart Before their Planned Departure Time

Minimum departure time for flight f: Γ_f

$$t_{(f,\,s)} \geq \Gamma_f$$

Sector Traversal Time is Fixed for Each Flight

$\Lambda_{(f,\,s)}$: Time flight f takes to traverse sector s

$$t_{(f,\,s+1)} - t_{(f,\,s)} = \Lambda_{(f,\,s)}$$

$$t_{(f,\,s_2)} - t_{(f,\,s_1)} = 10$$

No Sector May Be Overburdened

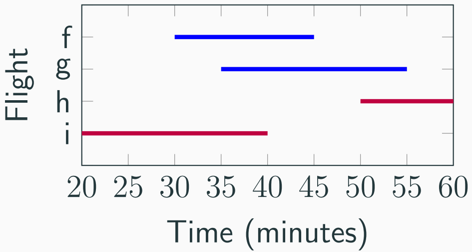
New Notation:

- c_s: The capacity c of sector s, given as limit on the number of flights at once in the sector.
- F_s: The set of flights that traverse s at some point.

The Hotspot Constraint

$x^s_{fg} = 1$ when flights f and g share sector s.

Sector s, where $c_s = 2$



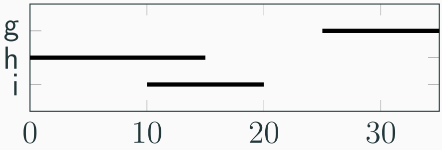
$|F_S| = 4$

$x^s_{fg} = 1$

$x^s_{hi} = 0$

The Hotspot Constraint

Sector s, where $c_s = 2$



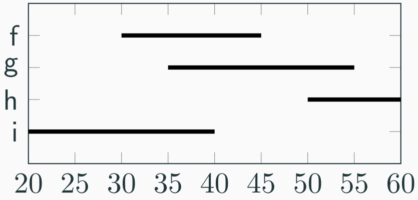
$$|F_S| = 3$$

$$x^s_{gh} + x^s_{gi} + x^s_{hi} \leq \binom{c_s + 1}{2} - 1$$

$$0 + 0 + 1 \leq \binom{3}{2} - 1$$

$$1 \leq 2$$

How is the Hotspot Constraint Broken?



$$|F_S| = 4$$

All pairwise combinations of $c_s + 1$ flights

$$x^s_{gh} + x^s_{gi} + x^s_{hi} \leq 2 \qquad 1 + 1 + 0 \leq 2$$

$$x^s_{fh} + x^s_{fi} + x^s_{hi} \leq 2 \qquad 0 + 1 + 0 \leq 2$$

$$\boldsymbol{x^s_{fg} + x^s_{fi} + x^s_{gi} \leq 2} \quad \rightarrow \quad \mathbf{1 + 1 + 1 \not\leq 2}$$

$$x^s_{fg} + x^s_{fh} + x^s_{gh} \leq 2 \qquad 1 + 0 + 1 \leq 2$$

The Order Constraints

- If flight f leaves s before g enters, $y^s_{fg} = 1$.
- If flight g leaves s before f enters, $y^s_{gf} = 1$.

Constraint: Only one binary quantity may be true for a pair of flights in given sector

$$y^s_{fg} + y^s_{gf} + x^s_{fg} = 1$$

The Big-M Conjunction

Remember...

$$y_{fg}^s + y_{gf}^s + x_{fg}^s = 1$$

Constraint:

$$
\begin{aligned}
\text{(i)} \quad & t_{(g,\,s)} - t_{(f,\,s+1)} \geq -M(1 - y_{fg}^s) \\
\text{(ii)} \quad & t_{(f,\,s)} - t_{(g,\,s+1)} \geq -M(1 - y_{gf}^s) \\
\text{(iii)} \quad & t_{(g,\,s+1)} - t_{(f,\,s)} \geq -M(1 - x_{fg}^s) \\
\text{(iv)} \quad & t_{(f,\,s+1)} - t_{(g,\,s)} \geq -M(1 - x_{fg}^s) \\
\text{where} \quad & y_{fg}^s,\ y_{gf}^s,\ x_{fg}^s \in \{0,\ 1\}
\end{aligned}
$$

Where M simulates ∞

The Big-M Conjunction

$$y^s_{fg} + y^s_{gf} + x^s_{fg} = 1$$
$$1 + 0 + 0 = 1$$

Constraint:

$$\begin{aligned}
\text{(i)} \quad & t_{(g,\,s)} - t_{(f,\,s+1)} \geq -M(1-1) \\
\text{(ii)} \quad & t_{(f,\,s)} - t_{(g,\,s+1)} \geq -M(1-0) \\
\text{(iii)} \quad & t_{(g,\,s+1)} - t_{(f,\,s)} \geq -M(1-0) \\
\text{(iv)} \quad & t_{(f,\,s+1)} - t_{(g,\,s)} \geq -M(1-0) \\
\text{where} \quad & y^s_{fg},\ y^s_{gf},\ x^s_{fg} \in \{0,\ 1\}
\end{aligned}$$

Where M simulates ∞

The Big-M Conjunction

$$y^s_{fg} + y^s_{gf} + x^s_{fg} = 1$$
$$1 + 0 + 0 = 1$$

Constraint:

$$\begin{aligned}
\text{(i)} \quad & t_{(g,\, s)} - t_{(f,\, s+1)} \geq 0 \\
\text{(ii)} \quad & t_{(f,\, s)} - t_{(g,\, s+1)} \geq -M \\
\text{(iii)} \quad & t_{(g,\, s+1)} - t_{(f,\, s)} \geq -M \\
\text{(iv)} \quad & t_{(f,\, s+1)} - t_{(g,\, s)} \geq -M \\
\text{where} \quad & y^s_{fg},\ y^s_{gf},\ x^s_{fg} \in \{0,\ 1\}
\end{aligned}$$

Where M simulates ∞

Graph

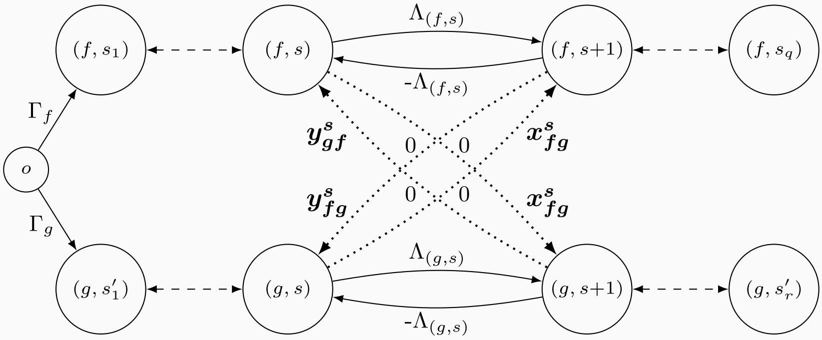


Image: Mannino and Sartor, 2018

Results

Experiments

- 400 simulated sectors
- From 100 up to 300 flights scheduled randomly
- 20 airports
- 30 simulation runs

Comparison of Graph and Big-M Constraint Formulations

	Flights	Hotspots	Visited Nodes		Time (s)	
			Graph	Big-M	Graph	Big-M
Median	199	6	116	2203	0.29	0.97
Max	296	15	3,625	172,950	1.35	50.61
Min	110	1	0	16,758	0.05	0.05

Summary of results across 30 trials.

References

[1] *Introduction to Algorithms*. eng. 3rd ed.. Cambridge, Mass.: MIT Press, 2009. ISBN: 9780262033848.

[2] Carlo Mannino, Andreas Nakkerud, and Giorgio Sartor. “Air traffic flow management with layered workload constraints”. In: *Computers and Operations Research* 127 (Mar. 2021), p. 105159. DOI: 10.1016/j.cor.2020.105159.

[3] Carlo Mannino and Giorgio Sartor. “The Path&Cycle Formulation for the Hotspot Problem in Air Traffic Management”. In: *18th Workshop on Algorithmic Approaches for Transportation Modelling, Optimization, and Systems (ATMOS 2018)*. Ed. by Ralf Borndörfer and Sabine Storandt. Vol. 65. OpenAccess Series in Informatics (OASIcs). Dagstuhl, Germany: Schloss Dagstuhl–Leibniz-Zentrum fuer Informatik, 2018, 14:1–14:11. ISBN: 978-3-95977-096-5. DOI: 10.4230/OASIcs.ATMOS.2018.14. URL: http://drops.dagstuhl.de/opus/volltexte/2018/9719.

Acknowledgments

Dr. Elena Machkasova, advisor

Brian Goslinga, draft feedback

Additional slides

Constraint

$$\sum_{\{f,g\}\subseteq K} x^s_{fg} \leq \binom{c_s + 1}{2} - 1$$

Where

$$K \subseteq F_s$$

$$|K| = c_s + 1$$

Linear Programming

- Concave feasible regions are impossible in linear programs.
- Local minimums are not necessarily global minimums in concave shapes.