Using Temporal Session Types to Analyze Time Complexities of Concurrent Programs

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Problem

Suppose you are the Morris Telegraph Company

You have a network of telegraph stations

- Each station sends, receives, and processes Morse code messages in various ways

How long does it take a message to get through the network?

Problem

Concurrent program —
A program with several parts running at the same time

Hard to tell how long a concurrent program will take to execute

- Many pieces interacting
  - Some can run in parallel
  - Some need to wait until other pieces are ready
- Tricky to figure out the timing of interactions

Need: a good way to work out the timing between pieces of concurrent programs

Solution

Das et al. (2018) give a way to analyze **the timing of interactions between parts of a program**.

- Big idea: adding timing information to datatypes
- Specifically, they introduce *temporal session types*
  - for describing channels of communication
  - “message rate” becomes part of the type system

Layers

- π-calculus
  - A simple, minimalist concurrent programming language
  - From 1992, developed by Milner et al.
- Session types
  - A way to typecheck π-calculus
  - From 1993, developed by Kohei Honda
- Temporal session types
  - Session types extended with timing information
  - From 2018, developed by Das et al.

Outline

- π-calculus
- Session types
- Temporal session types
- Conclusion

π-calculus

Session types

Temporal session types

π-calculus: motivation

Need a way to represent concurrent programs

Want it to be:

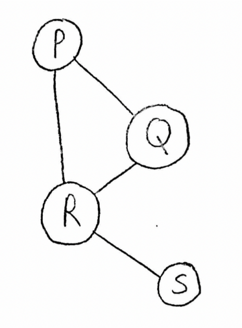
- general
- precise
- small

Several such systems exist

Das et al. use π*-calculus*

- From the early 1990s
- Good at modeling **independent processes that send messages** back and forth
  - servers on the web
  - processes in Unix

π-calculus: what is it?



What is π-calculus?

A very small programming language

Three main constructs:

- Processes
  - Like small programs
- Channels
- Labels
  - The data you send over channels
  - A finite set of symbols
  - E.g., for Morse code: $\{$ DOT, DASH, NEXT_LETTER, \$ $\}$

π-calculus: elementary operations

Defining a process

```
processName(channel1, channel2) =
   operation1;
   operation2;
   operation3;
   operation4
```

Spawning a process

```
a() =
   a();
   a()
```

π-calculus: elementary operations

Sending a label

```
sayHi(outChannel) =
  outChannel.DOT;
  outChannel.DOT;
  outChannel.DOT;
  outChannel.DOT;
  outChannel.NEXT_LETTER;
  outChannel.DOT;
  outChannel.DOT;
  outChannel.$;
  close outChannel
```

H I

•••• ••

The message “Hi” in Morse code

Receiving a label

```
invert(inChannel, outChannel) =
  case inChannel
  | DOT =>
      outChannel.DASH;
      invert(inChannel, outChannel)
  | DASH =>
      outChannel.DOT;
      invert(inChannel, outChannel)
  | NEXT_LETTER =>
      outChannel.NEXT_LETTER;
      invert(inChannel, outChannel)
  | $ =>
      outChannel.$;
      wait inChannel;
      close outChannel
```

π-calculus: what's next?

We have: a simple way to describe concurrent programs

Goal: figure out the times at which messages are sent over a channel

Next step: describe the interactions over a given channel

- This description is called the *session type* of the channel
- Eventually will include timing information
- But for now just says who's sending messages to whom

π-calculus

Session types

Temporal session types

Session types: what are they?

Session types are:

- Datatypes describing channels
- More complicated than `int` or `string`
- But they serve the same purpose:
  - Says what operations does this channel supports
- The typechecker makes sure you're using each channel correctly

Session types: what are they?

Session types describe **the structure of how two processes interact over a channel**

- E.g., “send two labels, then receive one label, then repeat”.

Like a very small network protocol

- A contract for how processes should talk to each other
- The typechecker makes sure you follow that contract

Session types: how are they written?

Internal choice

$$\oplus\{\ \mathtt{A} : T_1,\ \mathtt{B} : T_2\ \}$$

is a type meaning we can choose to:

- send label `A` and then do an action of type T_1
- send label `B` and then do an action of type T_2

Closing a channel

$$\mathbf{1}$$

is a type saying to close the channel immediately.

Session types: example

```
sayHi(output) =
  output.DOT;
  output.DOT;
  output.DOT;
  output.DOT;
  output.NEXT_LETTER;
  output.DOT;
  output.DOT;
  output.$;
  close output
```

$$
\textit{sendMessage} = \oplus\{
\quad \mathtt{DOT} : \textit{sendMessage},
\quad \mathtt{DASH} : \textit{sendMessage},
\quad \mathtt{NEXT\_LETTER} : \textit{sendMessage},
\quad \$: \mathbf{1}
\}
$$

The channel `output` has type *sendMessage*

Session types: how are they written?

External choice

$\&\{$

$\quad \mathtt{A} : T_1,$

$\quad \mathtt{B} : T_2$

$\}$

means we should be prepared to either:

- receive label $\mathtt{A}$ and then do an action of type T_1
- or receive label $\mathtt{B}$ and then do an action of type T_2

Waiting for a channel to close

$$\perp$$

means “wait for the other person to close this channel”.

Session types: example

```
invert(input, output) =
  case input
  | DOT =>
    output.DASH;
    invert(input, output)
  | DASH =>
    output.DOT;
    invert(input, output)
  | NEXT_LETTER =>
    output.NEXT_LETTER;
    invert(input, output)
  | $ =>
    output.$;
    wait input;
    close output
```

$$\mathit{sendMessage} = \oplus\{ \\ \quad \texttt{DOT} : \mathit{sendMessage}, \\ \quad \texttt{DASH} : \mathit{sendMessage}, \\ \quad \texttt{NEXT\_LETTER} : \mathit{sendMessage}, \\ \quad \$: \mathbf{1} \\ \}$$

$$\mathit{readMessage} = \&\{ \\ \quad \texttt{DOT} : \mathit{readMessage}, \\ \quad \texttt{DASH} : \mathit{readMessage}, \\ \quad \texttt{NEXT\_LETTER} : \mathit{readMessage}, \\ \quad \$: \bot \\ \}$$

The channel `input` has type *readMessage*
The channel `output` has type *sendMessage*

Session types: non-example

```
invert(input, output) =
  case input
  | DOT =>
      output.DASH;
      invert(input, output)
  | NEXT_LETTER =>
      output.NEXT_LETTER;
      invert(input, output)
  | $ =>
      output.$;
      wait input;
      close output
```

$$\mathit{sendMessage} = \oplus\{\ \texttt{DOT} : \mathit{sendMessage},\ \texttt{DASH} : \mathit{sendMessage},\ \texttt{NEXT\_LETTER} : \mathit{sendMessage},\ \$: \mathbf{1}\ \}$$

$$\mathit{readMessage} = \&\{\ \texttt{DOT} : \mathit{readMessage},\ \texttt{DASH} : \mathit{readMessage},\ \texttt{NEXT\_LETTER} : \mathit{readMessage},\ \$: \bot\ \}$$

The channel `input` **does not** have type *readMessage*
The channel `output` has type *sendMessage*

Session types: what's next?

We have: the structure of interactions over a channel

Goal: figure out the times at which those interactions happen

Next step: add timing information to session types

π-calculus

Session types

Temporal session types

Temporal session types: what are they?

New session type: **delay**

$$\circ T$$

means that an action of type T will occur after one second

Temporal session types: example

Each I/O operation takes one second

By convention, delays occur after the operation

```
sayHi(output) =
  output.DOT;               (delay 1)
  output.DOT;               (delay 1)
  output.DOT;               (delay 1)
  output.DOT;               (delay 1)
  output.NEXT_LETTER;       (delay 1)
  output.DOT;               (delay 1)
  output.DOT;               (delay 1)
  output.$;                 (delay 1)
  close output              (delay 1)
```

$$
\mathit{timedSendMessage} = \oplus\{
\quad \texttt{DOT} : \circ\mathit{timedSendMessage},
\quad \texttt{DASH} : \circ\mathit{timedSendMessage},
\quad \texttt{NEXT\_LETTER} : \circ\mathit{timedSendMessage},
\quad \$: \circ\mathbf{1}
\}
$$

The channel `output` has type *timedSendMessage*

One "$\circ$" in *timedSendMessage*, so
message rate = one label per second

Temporal session types: example

What about `invert(input, output)`?

```
invert(input, output) =
  case input
  | DOT =>
      output.DASH;
      invert(input, output)
  | DASH =>
      output.DOT;
      invert(input, output)
  | NEXT_LETTER =>
      output.NEXT_LETTER;
      invert(input, output)
  | $ =>
      output.$;
      wait input;
      close output
```

- Slightly harder
- Issue: timing of `output` will depend on timing of `input`
- But, if we know the timing of `input`, we can find the timing of `output`

Temporal session types: example

```
invert(input, output) =
  case input
  | DOT =>
      output.DASH;
      invert(input, output)
  | DASH =>
      output.DOT;
      invert(input, output)
  | NEXT_LETTER =>
      output.NEXT_LETTER;
      invert(input, output)
  | $ =>
      output.$;
      wait input;
      close output
```

Suppose we know that `input` has this temporal session type:

$$
\begin{aligned}
&\mathit{readMessageSlowly} = \&\{\\
&\quad \texttt{DOT} : \circ^n \mathit{readMessageSlowly},\\
&\quad \texttt{DASH} : \circ^n \mathit{readMessageSlowly},\\
&\quad \texttt{NEXT\_LETTER} : \circ^n \mathit{readMessageSlowly},\\
&\quad \$: \circ^n \bot\\
&\}
\end{aligned}
$$

Here, $\circ^n$ is a delay of n seconds.

Temporal session types: example

```
invert(input, output) =
  case input
  | DOT =>                     (delay 1)
      output.DASH;             (delay 1)
                               (delay k)
      invert(input, output)
  | DASH =>                    (delay 1)
      output.DOT;              (delay 1)
                               (delay k)
      invert(input, output)
  | NEXT_LETTER =>             (delay 1)
      output.NEXT_LETTER;      (delay 1)
                               (delay k)
      invert(input, output)
  | $ =>                       (delay 1)
      output.$;                (delay 1)
                               (delay k)
      wait input;              (delay 1)
      close output             (delay 1)
```

$$
\begin{aligned}
&\textit{readMessageSlowly} = \&\{\\
&\quad \texttt{DOT} : \circ^{n}\,\textit{readMessageSlowly},\\
&\quad \texttt{DASH} : \circ^{n}\,\textit{readMessageSlowly},\\
&\quad \texttt{NEXT\_LETTER} : \circ^{n}\,\textit{readMessageSlowly},\\
&\quad \$: \circ^{n}\,\bot\\
&\}
\end{aligned}
$$

Can sketch out where delays are:

- 1 second after each I/O operation
- k seconds where `invert` is idling
- Note: spawning a new process is instantaneous

Temporal session types: example

```
invert(input, output) =
  case input
  | DOT =>                      (delay 1)
      output.DASH;              (delay 1)
                                (delay k)
      invert(input, output)
  | DASH =>                     (delay 1)
      output.DOT;               (delay 1)
                                (delay k)
      invert(input, output)
  | NEXT_LETTER =>              (delay 1)
      output.NEXT_LETTER;       (delay 1)
                                (delay k)
      invert(input, output)
  | $ =>                        (delay 1)
      output.$;                 (delay 1)
                                (delay k)
      wait input;               (delay 1)
      close output              (delay 1)
```

$$
\begin{aligned}
&\mathit{readMessageSlowly} = \&\{ \\
&\quad \texttt{DOT} : \circ^{n}\mathit{readMessageSlowly}, \\
&\quad \texttt{DASH} : \circ^{n}\mathit{readMessageSlowly}, \\
&\quad \texttt{NEXT\_LETTER} : \circ^{n}\mathit{readMessageSlowly}, \\
&\quad \$: \circ^{n}\bot \\
&\}
\end{aligned}
$$

Delay between successive reads must equal n

Solve for k in terms of n:

$$1 + 1 + k = n$$
$$k = n - 2$$

Temporal session types: example

```
invert(input, output) =
  case input
  | DOT =>                    (delay 1)
      output.DASH;            (delay 1)
                              (delay k)
      invert(input, output)
  | DASH =>                   (delay 1)
      output.DOT;             (delay 1)
                              (delay k)
      invert(input, output)
  | NEXT_LETTER =>            (delay 1)
      output.NEXT_LETTER;     (delay 1)
                              (delay k)
      invert(input, output)
  | $ =>                      (delay 1)
      output.$;               (delay 1)
                              (delay k)
      wait input;             (delay 1)
      close output            (delay 1)
```

$$k = n - 2$$

What next?

k can't be a negative amount of time. So,

$$\begin{aligned} k &\geq 0 \\ n - 2 &\geq 0 \\ n &\geq 2. \end{aligned}$$

So, there must be at least 2 seconds between inputs.

(Otherwise, `invert` won't be able to read fast enough.)

Temporal session types: example

```
invert(input, output) =
  case input
  | DOT =>                  (delay 1)
    output.DASH;            (delay 1)
                            (delay k)
    invert(input, output)
  | DASH =>                 (delay 1)
    output.DOT;             (delay 1)
                            (delay k)
    invert(input, output)
  | NEXT_LETTER =>          (delay 1)
    output.NEXT_LETTER;     (delay 1)
                            (delay k)
    invert(input, output)
  | $ =>                    (delay 1)
    output.$;               (delay 1)
                            (delay k)
    wait input;             (delay 1)
    close output            (delay 1)
```

What next?

Find the temporal session type of *output*

- Initial delay: 1 second
- Delay between writes:

$$1 + k + 1 = n \text{ seconds}$$

Temporal session type:
$\circ\mathit{sendMessageSlowly}$

where

$$\mathit{sendMessageSlowly} = \oplus\{ \\ \texttt{DOT} : \circ^n \mathit{sendMessageSlowly}, \\ \texttt{DASH} : \circ^n \mathit{sendMessageSlowly}, \\ \texttt{NEXT\_LETTER} : \circ^n \mathit{sendMessageSlowly}, \\ \$: \circ^n \mathbf{1} \\ \}$$

Temporal session types: example

```
invert(input, output) =
  case input
  | DOT =>                        (delay 1)
      output.DASH;                (delay 1)
                                  (delay k)
      invert(input, output)
  | DASH =>                       (delay 1)
      output.DOT;                 (delay 1)
                                  (delay k)
      invert(input, output)
  | NEXT_LETTER =>                (delay 1)
      output.NEXT_LETTER;         (delay 1)
                                  (delay k)
      invert(input, output)
  | $ =>                          (delay 1)
      output.$;                   (delay 1)
                                  (delay k)
      wait input;                 (delay 1)
      close output                (delay 1)
```

In summary:

Maximum message rate of `input`:
  2 labels/second

Message rate of `output`:
  n labels/second
  (same as `input`)

Latency of `output`:
  1 second

Conclusion

What do we have?

1. A way to find the timing of interactions between parts of a concurrent program
2. A way to mechanically verify that the timing is correct
   - Given:
     - π-calculus source code
     - a temporal session type for each channel

   a typechecker can verify the channels actually have the session types indicated.

What we don't have (yet):

1. Implementations
2. A way to deduce temporal session types from source code

Questions?

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