Post Quantum Cryptography

Lattice Based Algorithms

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CSci Seminar Fall 2025

Post Quantum Cryptography Lattice Based Algorithms

- Rapid development of quantum computers that are exponentially faster and stronger than what there is today.
- Data that is shared over the internet is not safe. Messages, banking, private data will be compromised
- 1994 Peter Shor proved that with a strong enough quantum computer, all current cryptography can be broken
- NIST has started developing the future of security with post quantum cryptography

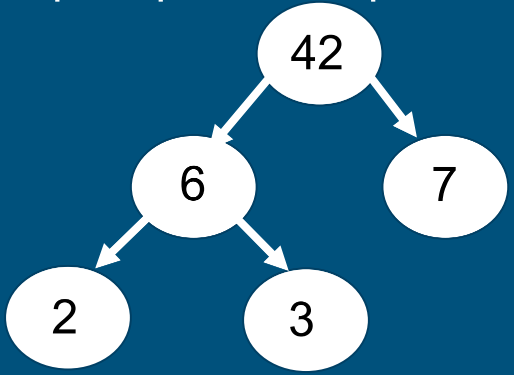
Overview

- Current algorithms
- Shor’s algorithm
- NIST
- Post Quantum Cryptography
- Lattices
- Learning With Errors
- Kyber
- SABER
- Conclusion

Current algorithms

- RSA named after Rivest–Shamir–Adleman the computer scientist who developed it
- RSA relies heavily on integer factorization
- A classical computer would take hundreds of trillions of years to break standard RSA encryption

Given a number N, the goal is to find prime numbers p1,p2,... ,pk such that N=p1✕ p2 ✕ …✕ pk



Current algorithms

- Elliptic Curve Cryptography (ECC) relies on discrete logarithm problem.
- Classic computer would take an astronomically long time to break ECC, it would take billions of years to break

$$x \text{ in } g^x \equiv h \pmod{\text{p}}$$

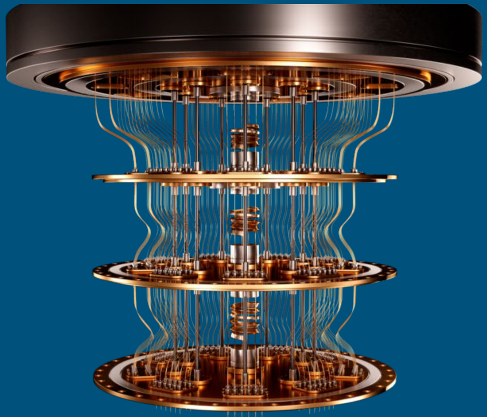
$$2^x \equiv 9 \pmod{11}$$

Current algorithms

- Major companies IBM, Google, and Microsoft are developing quantum computers
- Quantum computer and Shor’s algorithm will break current methods of encryption making them extremely less effective at protecting data

Quantum computers

- Quantum Bits (qubits) can represent 1, 0, or both at the same time this is called superposition
- Entanglement links qubits so they are mathematically correlated, allowing the computer to explore many possibilities in parallel
- Quantum parallelism



Shor’s algorithm

- Peter Shor developed an algorithm that could efficiently break the cryptographic systems used today.
- Shor’s algorithm is described as a hybrid algorithm, combining classical and quantum steps.

Pick random Integer A that A < N

Find the period R of f(x) = A^x mod N

If R is even and A^(R/2) is not -1(mod N), find the greatest common divisors of A^(R/2)± 1 and N

These will be non trivial factors of N

If Process fails try a different A

NIST

NIST
National Institute of
Standards and Technology

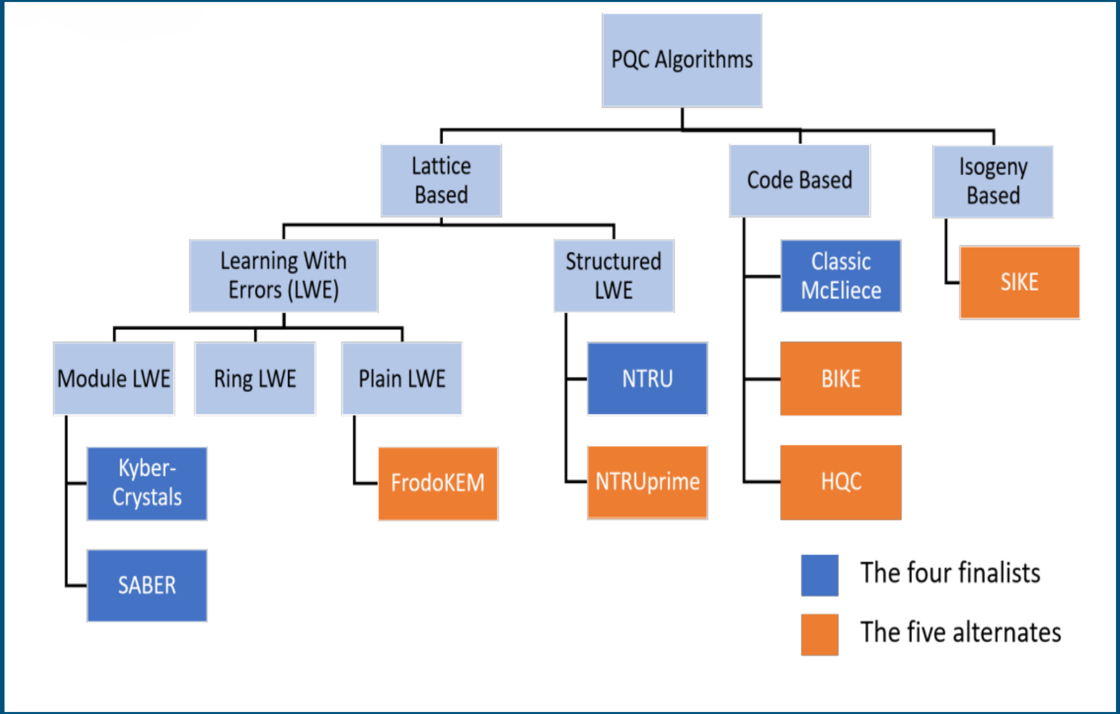
- U.S. federal agency within the Department of Commerce
- Founded on March 3, 1901, as the National Bureau of Standards. It was renamed to the National Institute of Standards and Technology in 1988
- Promotes innovation and industrial competitiveness through measurements of science, standards, and technology.

NIST process

- Identify the problem
- Call for Proposals
- Submission and Initial Evaluation
- Multi -Round Evaluation
- Selection of Finalists
- Standardization and Finalization

Post-Quantum Cryptography

- NIST finalists for PQC
- Code based
- Isogeny based
- Lattice based



PQC Algorithms

Lattice Based

Learning With Errors (LWE)

Module LWE

Kyber-Crystals

SABER

Ring LWE

Plain LWE

FrodoKEM

Structured LWE

NTRU

NTRUprime

Code Based

Classic McEliece

BIKE

HQC

Isogeny Based

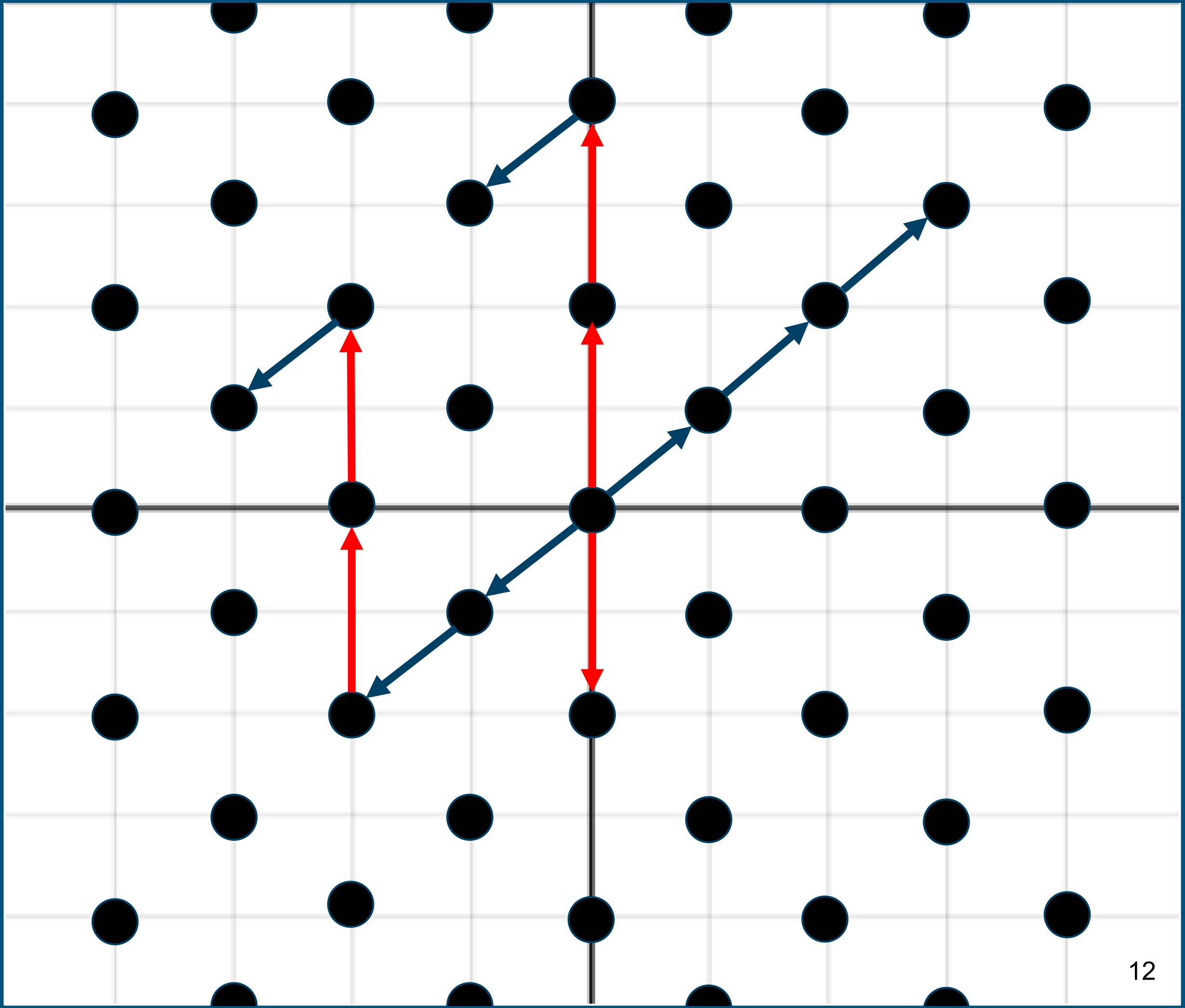
SIKE

🟦 The four finalists

🟧 The five alternates

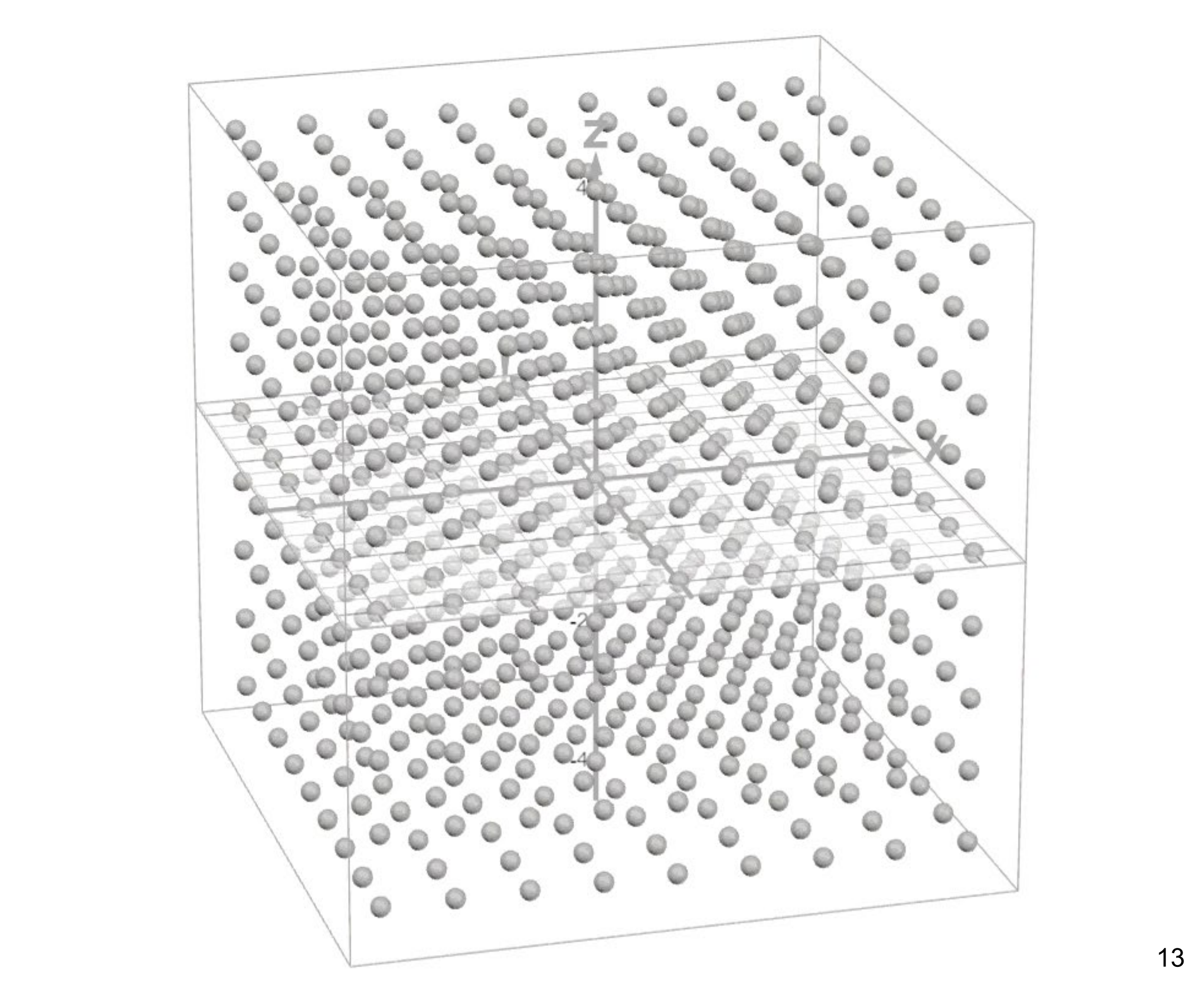
Lattices

- A lattice is a grid like structure with a repeating pattern
- The basis vectors are (0,2) and (1,1)



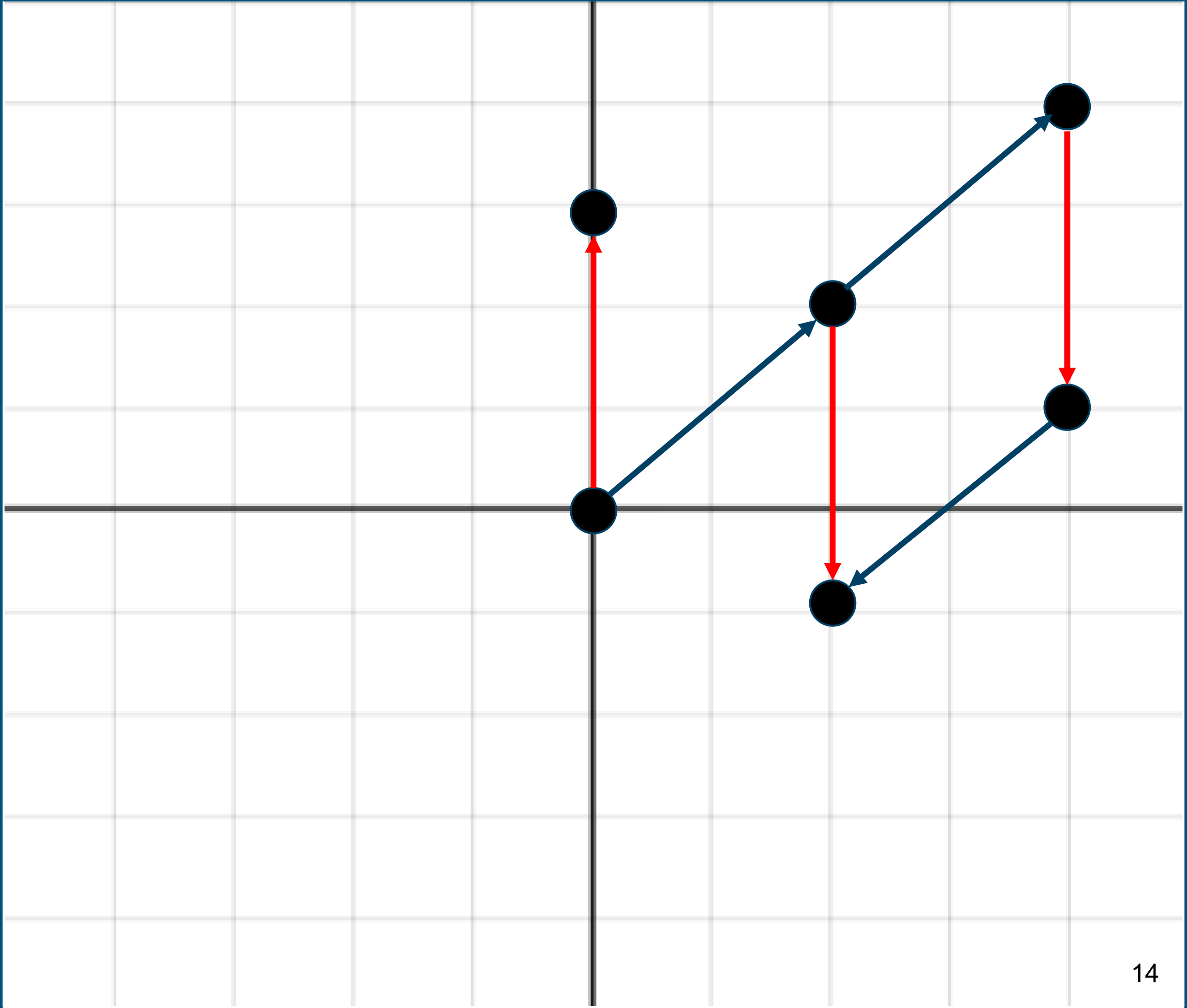
Lattices

- Each dimension adds more complexity



Lattices

- Shortest Vector Problem
- How to find the shortest vector that isn't the zero vector?



Learning with Errors

- Given a set of equations with small random errors find the hidden secret vector
- These are the subcategories of LWE: Plain LWE, Ring LWE, and Module LWE

Plain LWE

$$\begin{array}{r|l|}
77x + 7y + 28z + 23w = 2859 & -1 \\
21x + 19y + 30z + 48w = 3508 & +3 \\
4x + 24y + 33z + 38w = 3848 & -2 \\
8x + 20y + 84z + 61w = 6225 & +0
\end{array}$$

s

x= 10

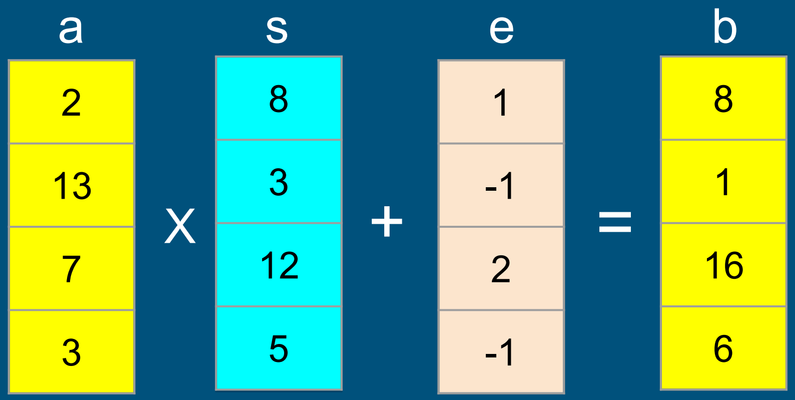
y=82

z=50

w=5

Ring Learning with Errors

- Used to build efficient and secure lattice-based cryptographic schemes
- Operates over the ring R, Let R=Zq[x]/(x^n)+1



$$\begin{array}{ccccccc} a & & s & & e & & b \\ 2 & & 8 & & 1 & & 8 \\ 13 & & 3 & & -1 & & 1 \\ 7 & X & 12 & + & 2 & = & 16 \\ 3 & & 5 & & -1 & & 6 \end{array}$$

Module Learning with Errors

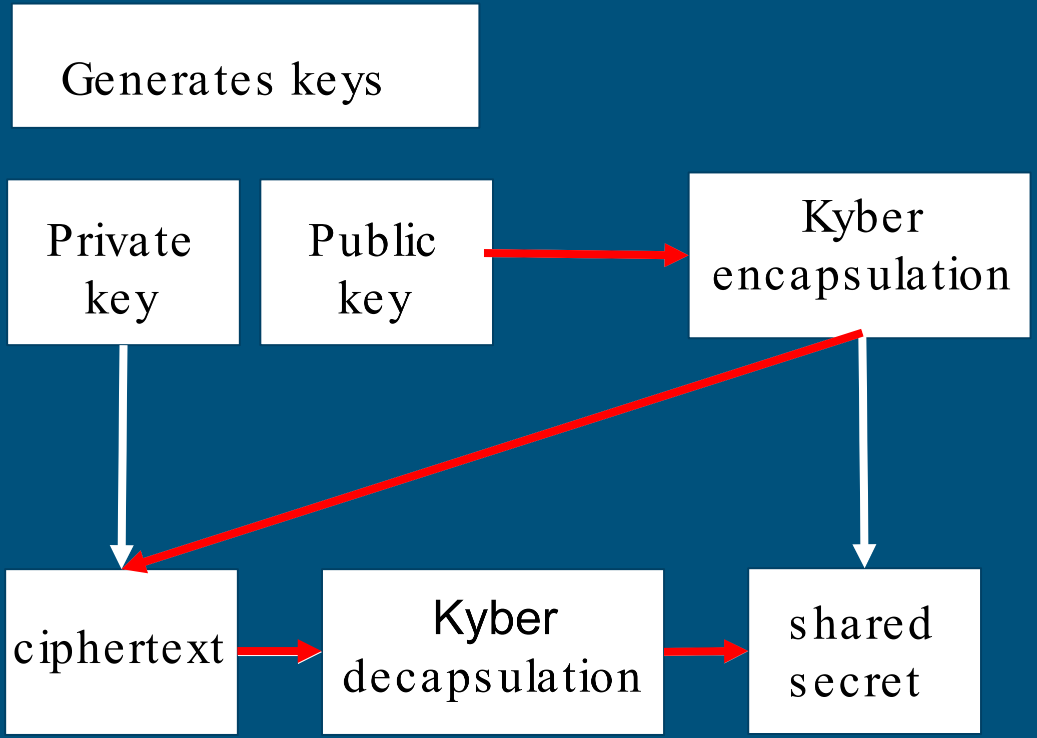
- Finding a secret module vector s given a set of "noisy" linear equations.
- equations are formed by taking inner products of known module vectors a_i with the secret s, and then adding a small, randomly distributed error e_i.

- secret vector s
- a_i are known random vectors
- e_i are small errors
- b_i is the result
- multiple equations of the form:

$$a_i * s + e_i \approx b_i \pmod{q}$$

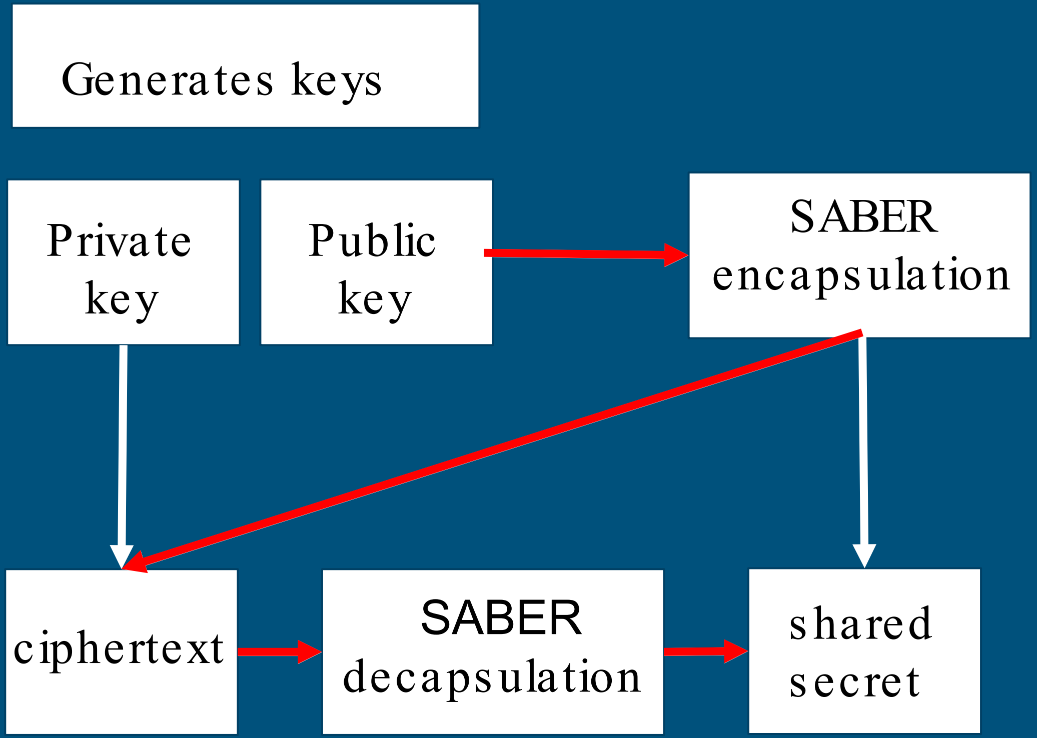
Kyber

- Encryption scheme based on module-LWE
- Built for speed and compactness
- Kyber 512, Kyber 768, Kyber 1024



SABER

- Built for simplicity and resilience
- LightSABER, SABER, FireSABER



Generates keys

Private key

Public key

SABER encapsulation

ciphertext

SABER decapsulation

shared secret

Conclusion

- Quantum computer are coming
- Shor’s algorithm proves this isn’t theory
- PQC is the solution
- Lattice based schemes like Kyber and SABER are leading the way
- The transition to PQC will take time, but the groundwork is being laid now.
- Early adoption ensures long-term security

References

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