Alpha-Beta Pruning in Chess Engines

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1997: Kasparov vs Deep Blue

https://espnfivethirtyeight.files.wordpress.com/2014/10/51659306crop.jpg?w=1824

Outline

- How Chess Engines think
- Search Trees
- Minimax
- Alpha-Beta
- Implementations in Chess Engines
- Enhancements

How Chess Engines think

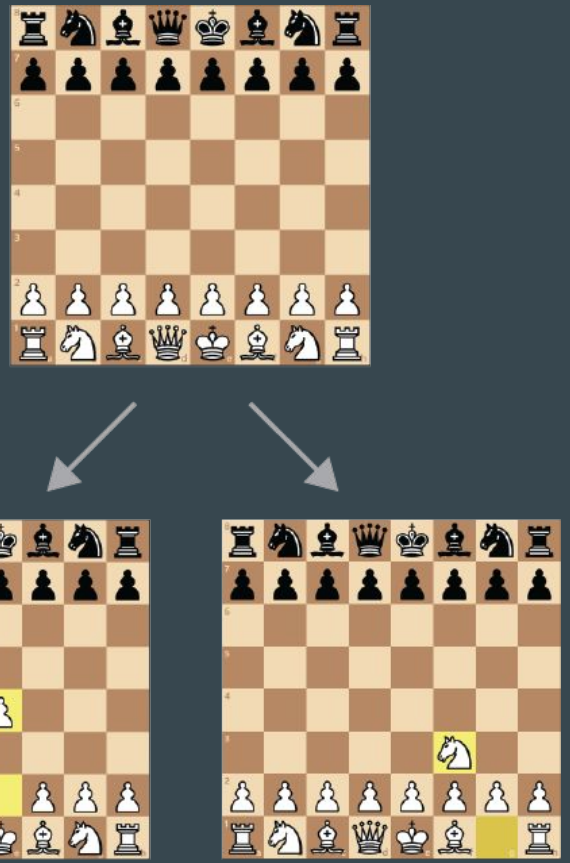
Ways to play chess

- Analysis, strategy, experience
- Possible moves
- Look at possible moves and evaluate
- Look ahead as far as we can and evaluate
- ~~Solving chess, brute force~~

Search Trees

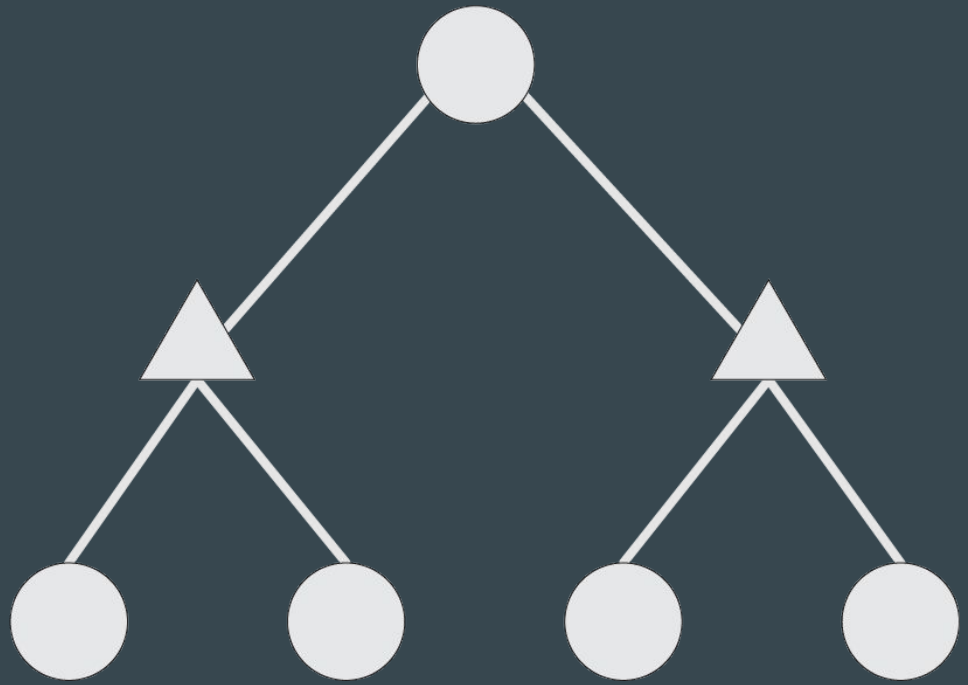
Search Tree

- The tree is created by each board state one move apart
- Each node in a branch represents alternating white and black moves
- $V = c_1f_1+c_2f_2+...+c_nf_n$
- c_i = value of one feature of the board
- f_i = function for how important the c_i is



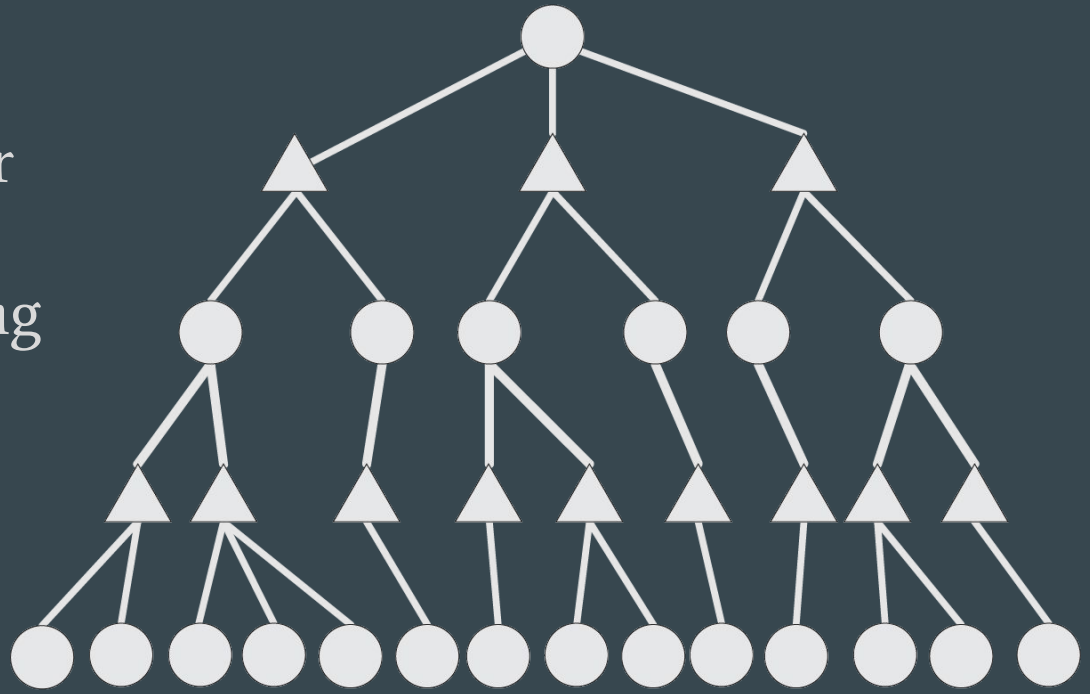
Adversarial Search Trees

- The adversarial search tree consists of nodes which captures all the possible moves in the game
- Each move is represented in terms of loss and gain for one of the players
- Move of Max player is a circle
- Move of Min player is a triangle



Adversarial Search Tree

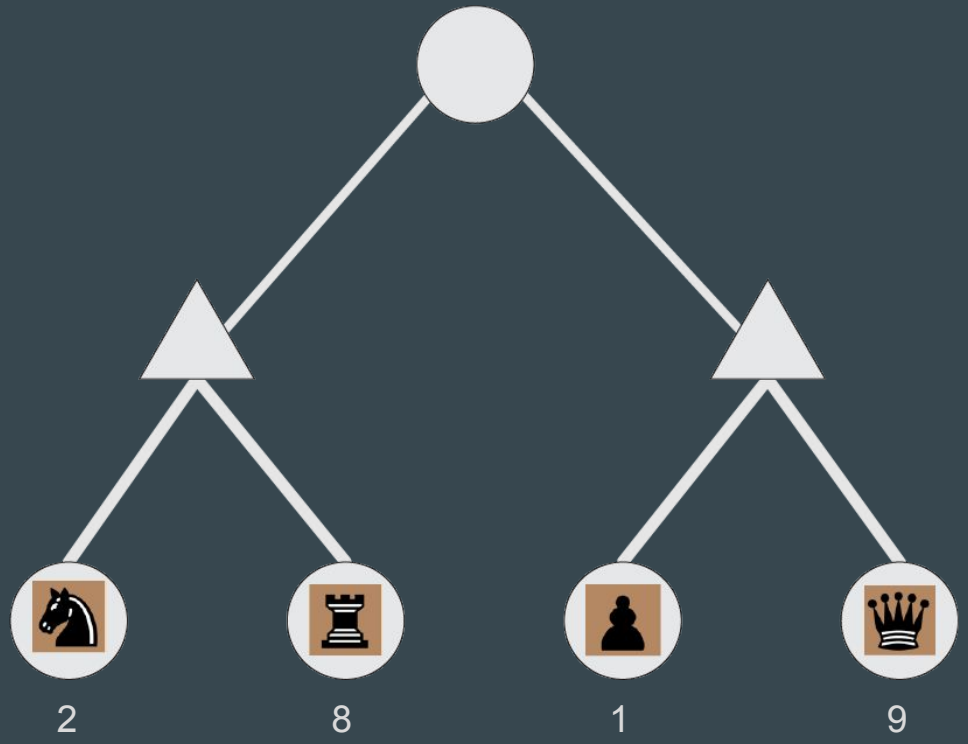
- Used for multiple player games
- Alternates who is making a move
- Chess is a two-player, zero-sum, perfect information game



Minimax

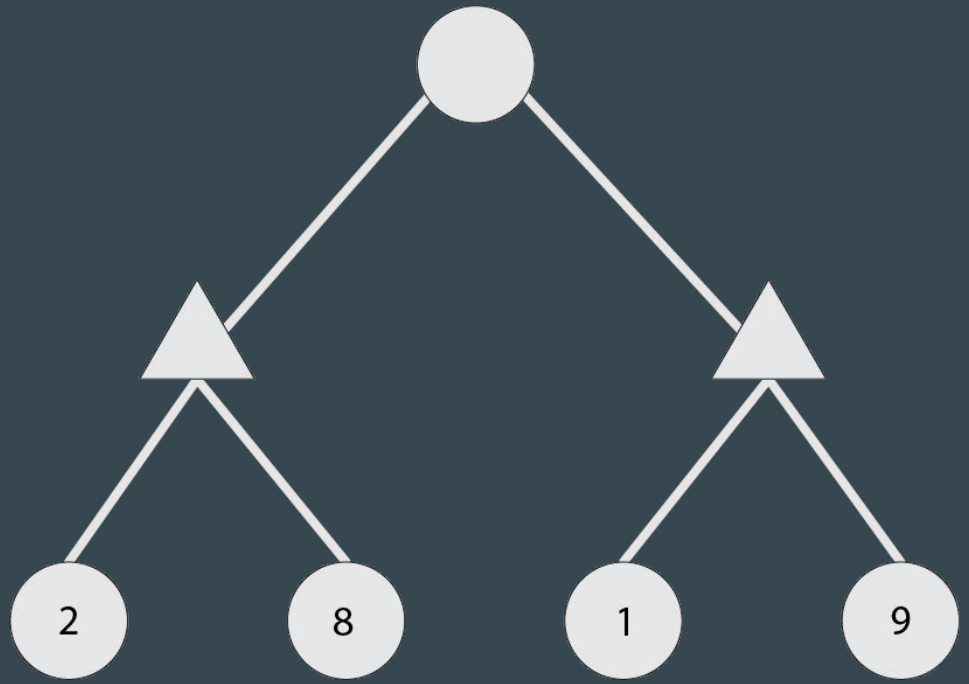
Minimax

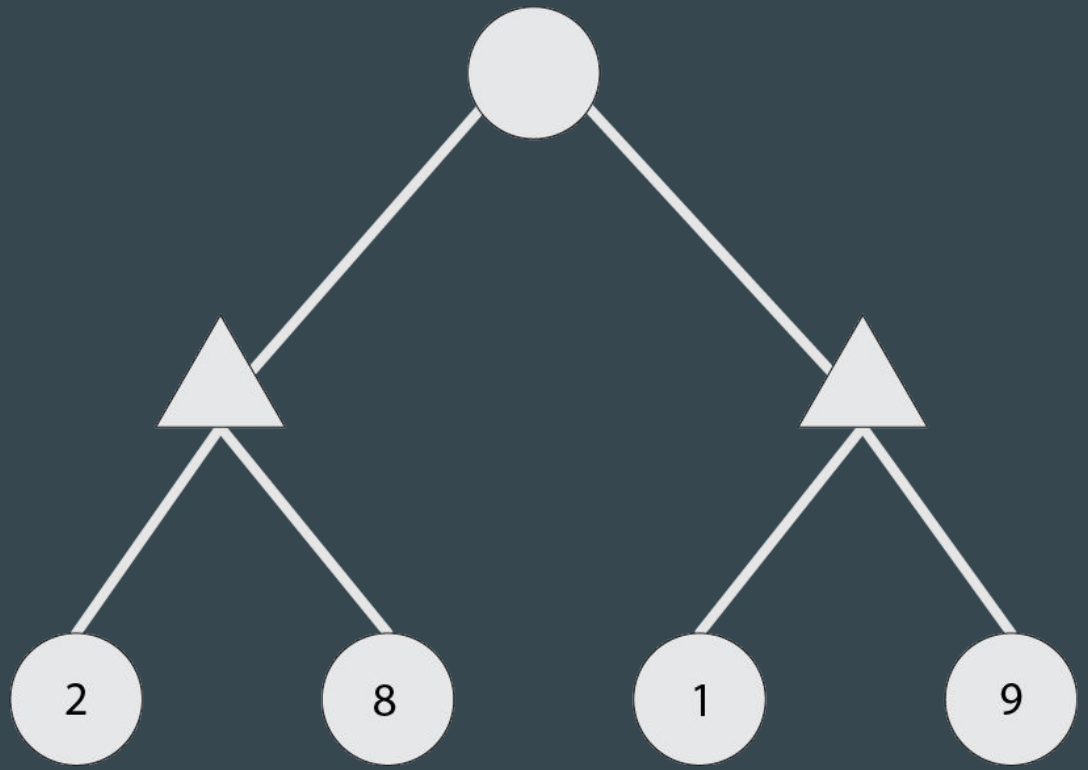
- The minimax algorithm is a decision rule used to minimize the loss for the worst possible case
- It returns the choice the highest guaranteed value without knowing the moves of the other players



Minimax

- Uses the adversarial search tree
- Space complexity of $O(b^m)$
- b = max branching factor of the search tree
- m = maximum depth of the search space
- Chess is a finite game

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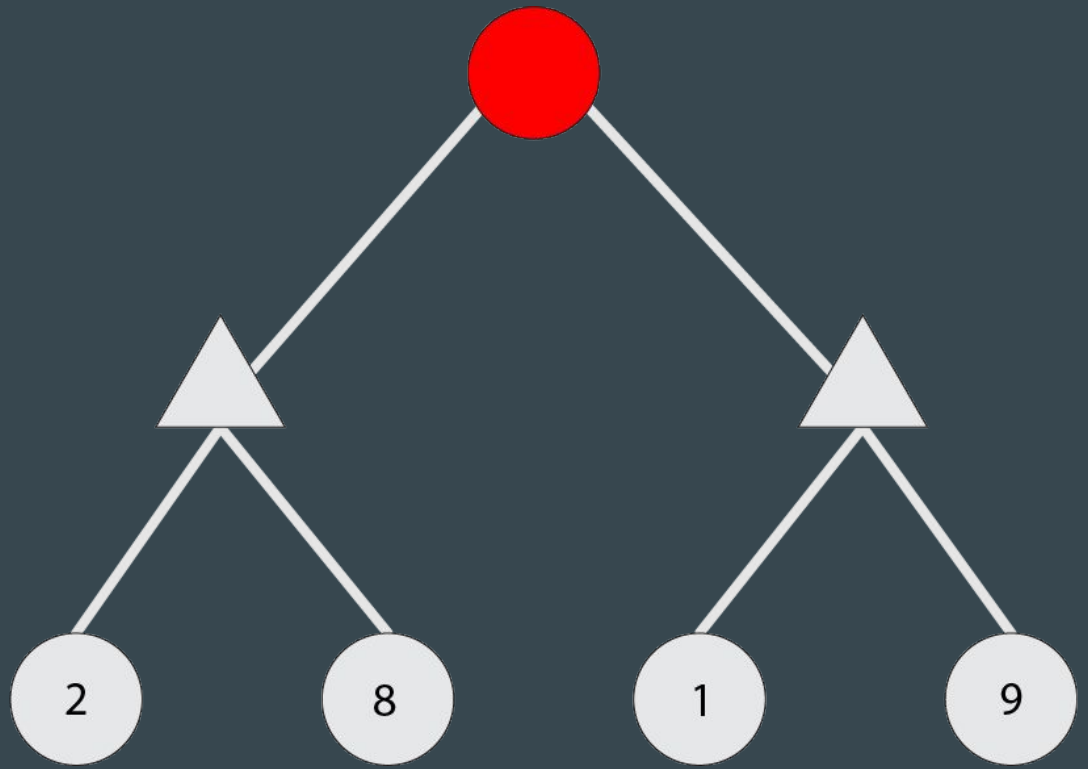
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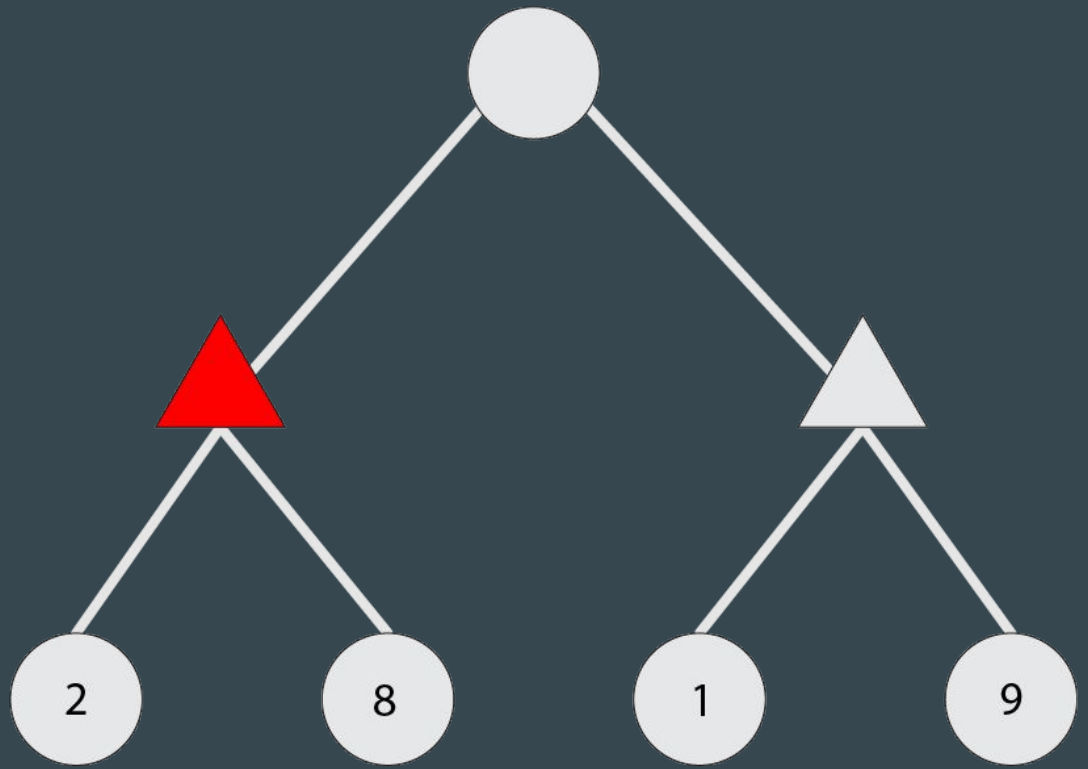
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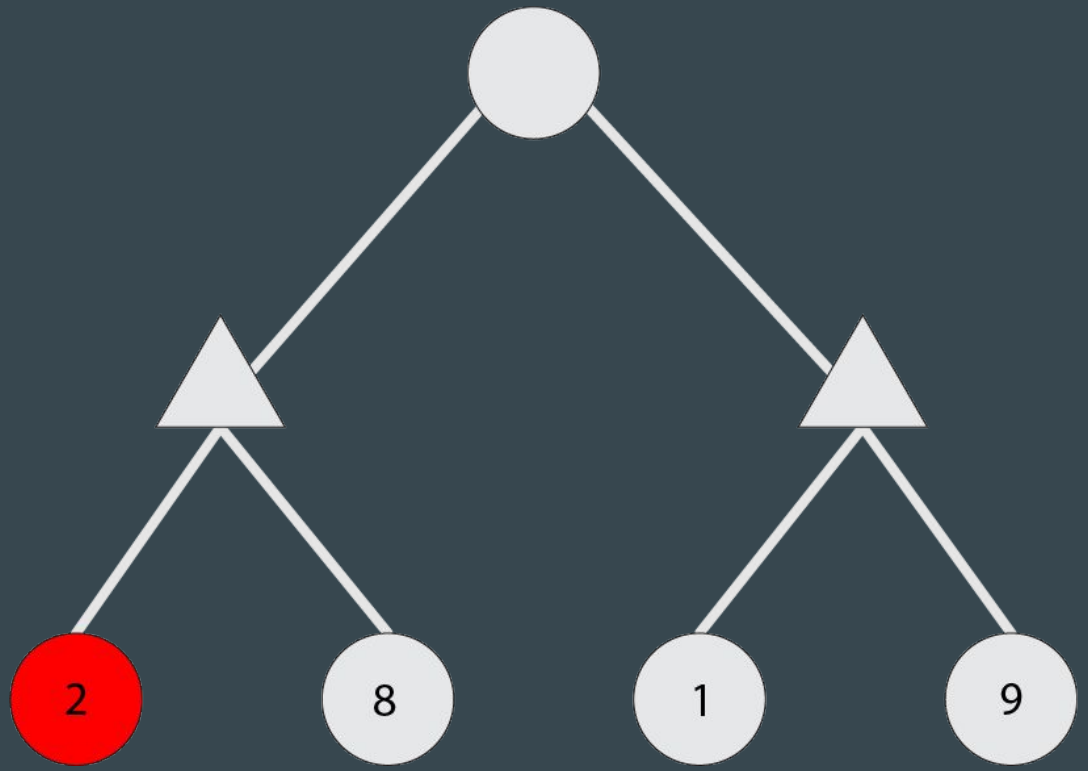
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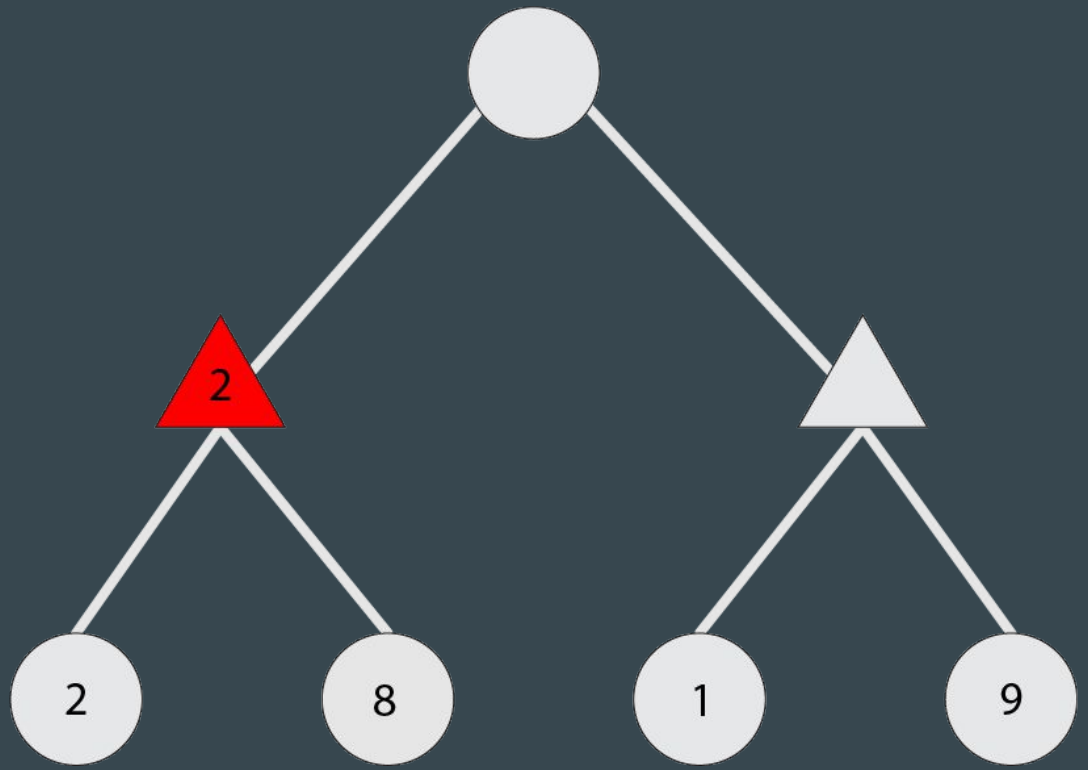
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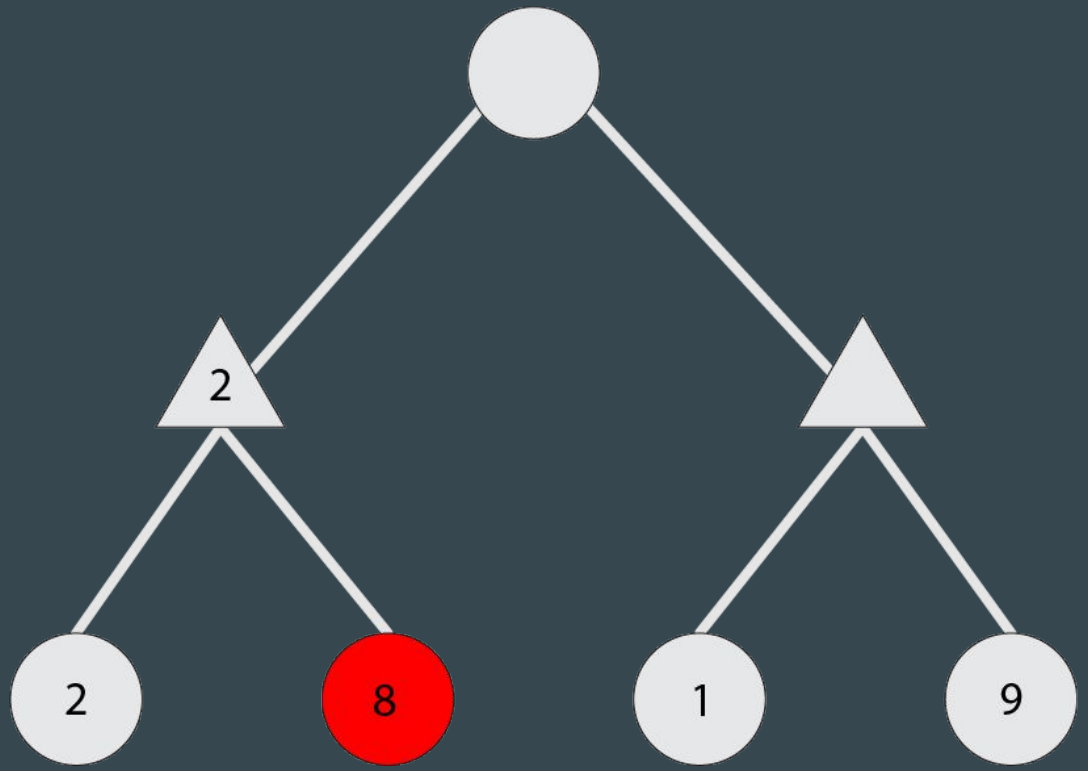
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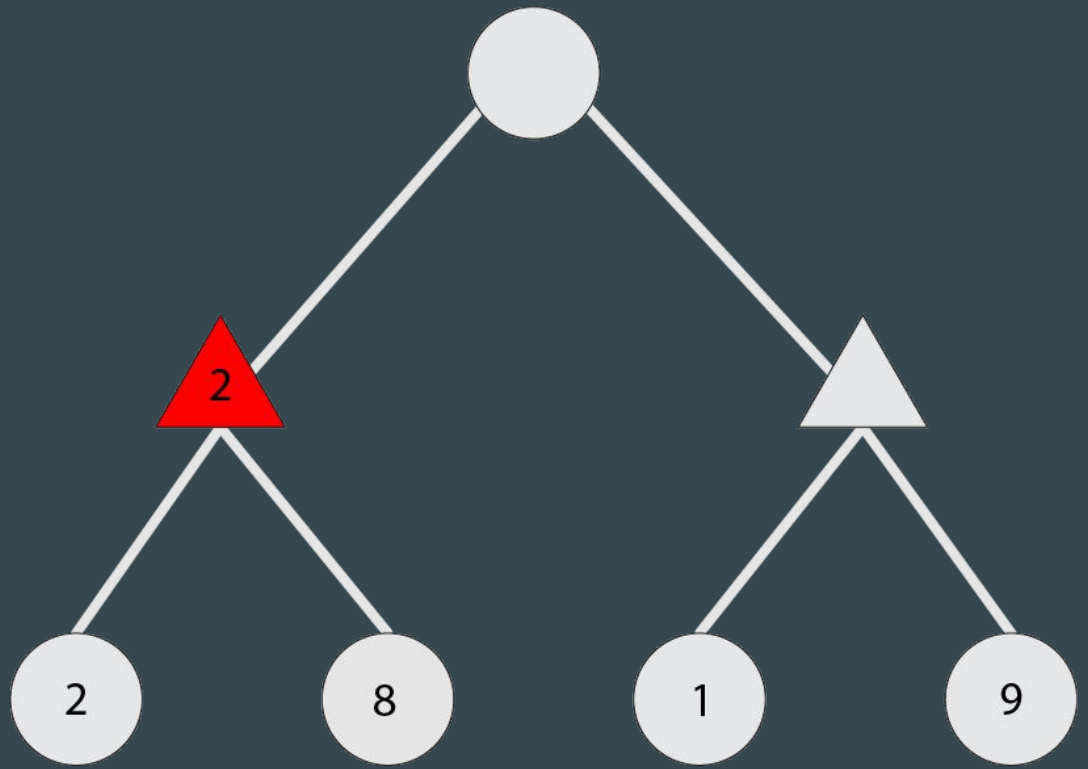
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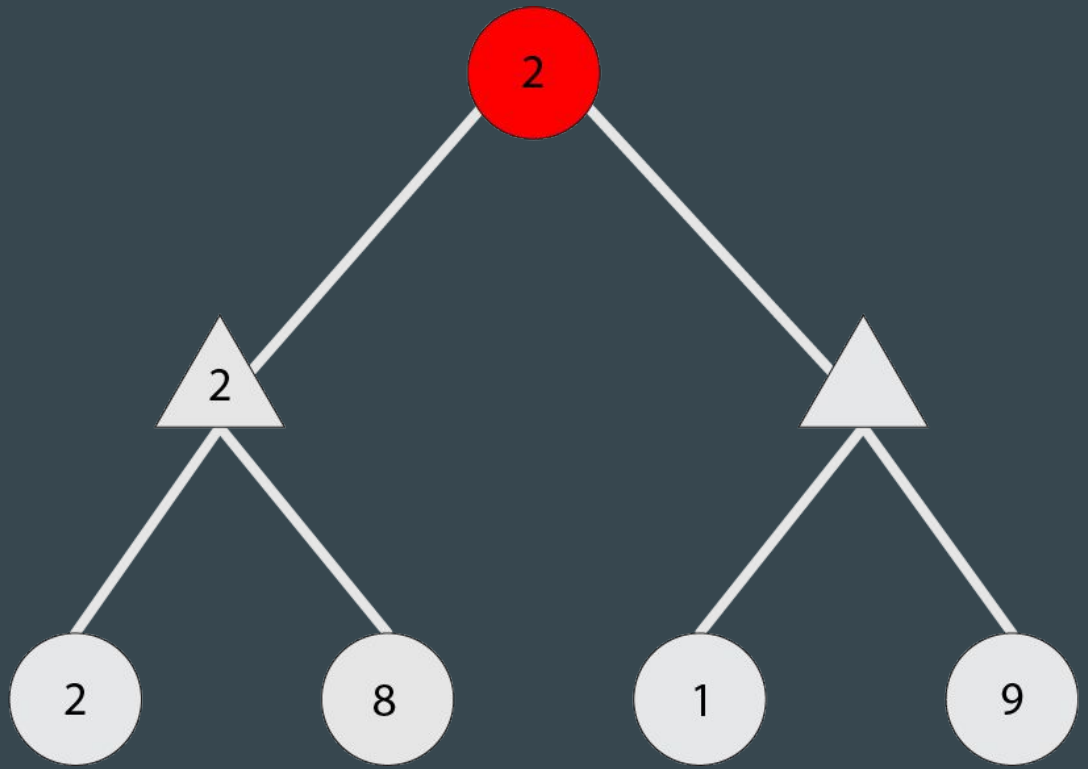
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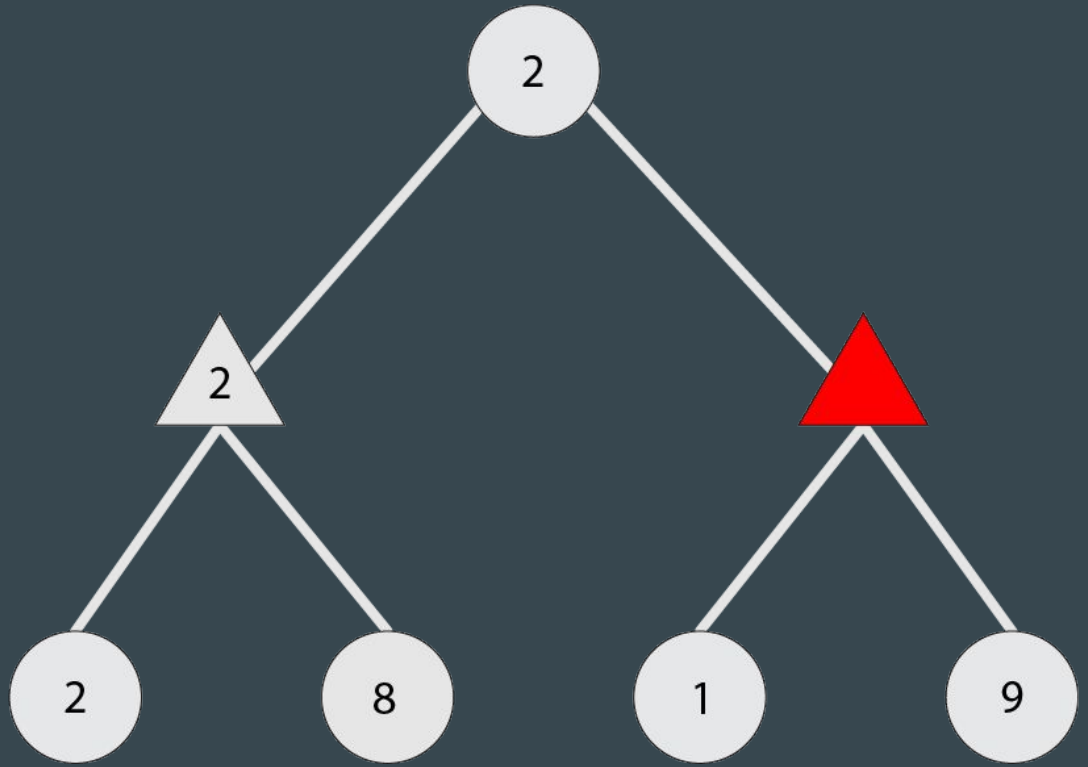
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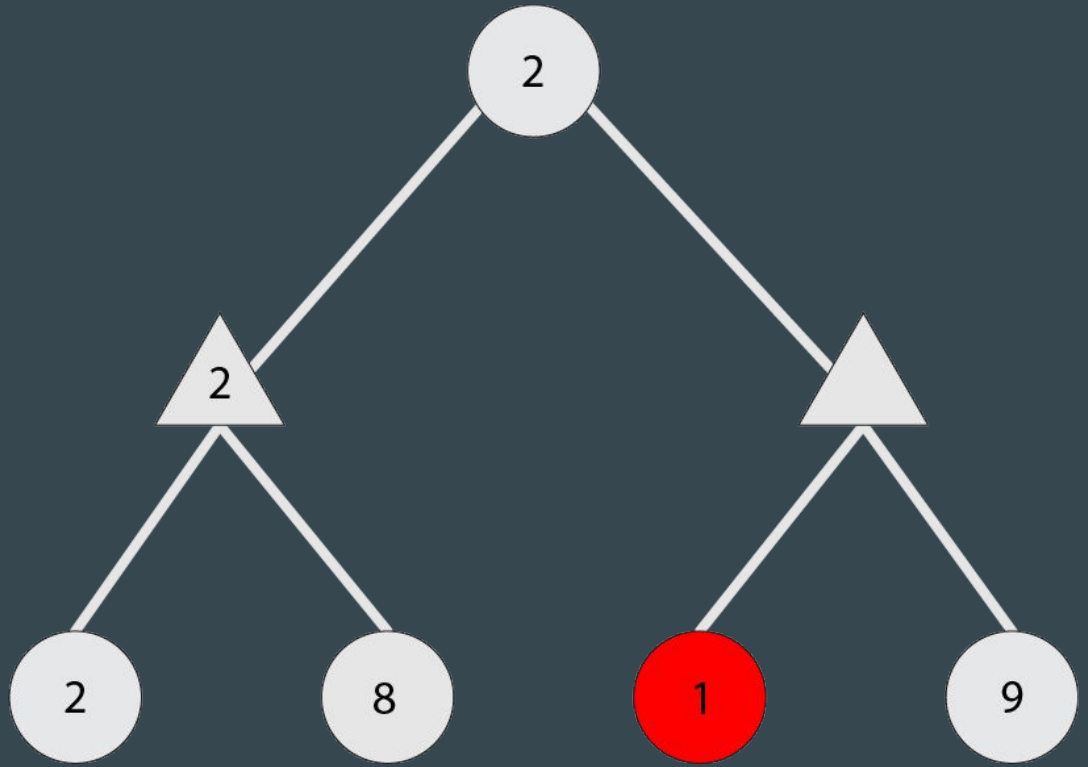
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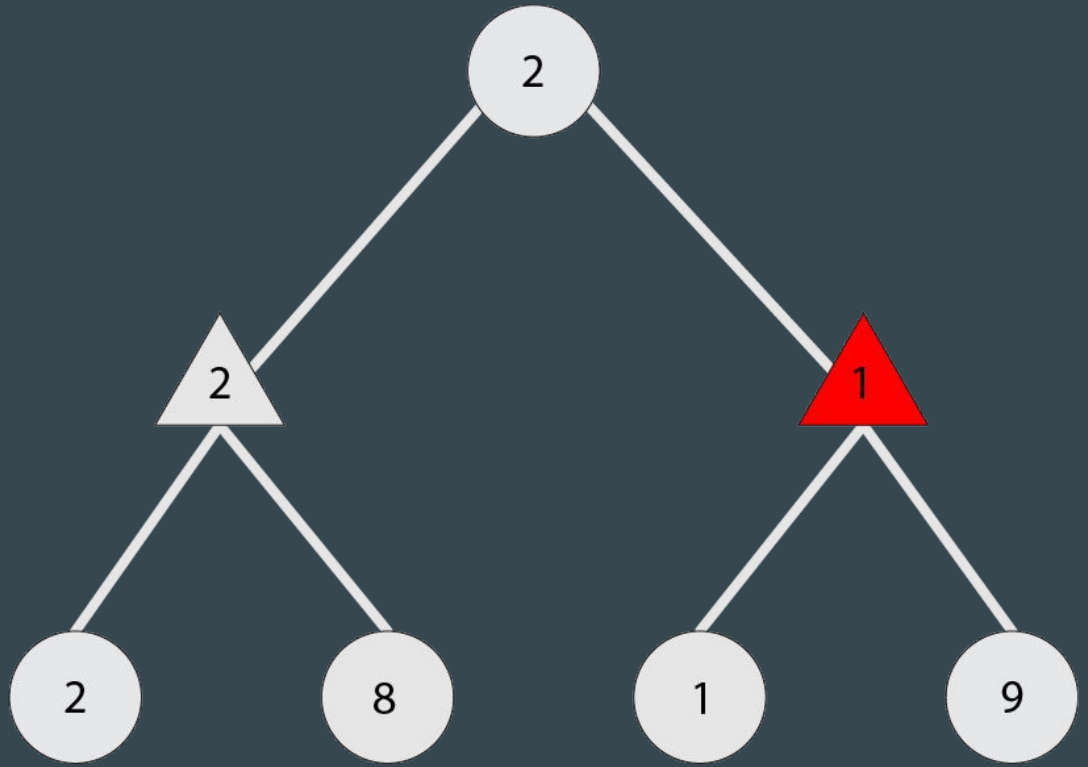
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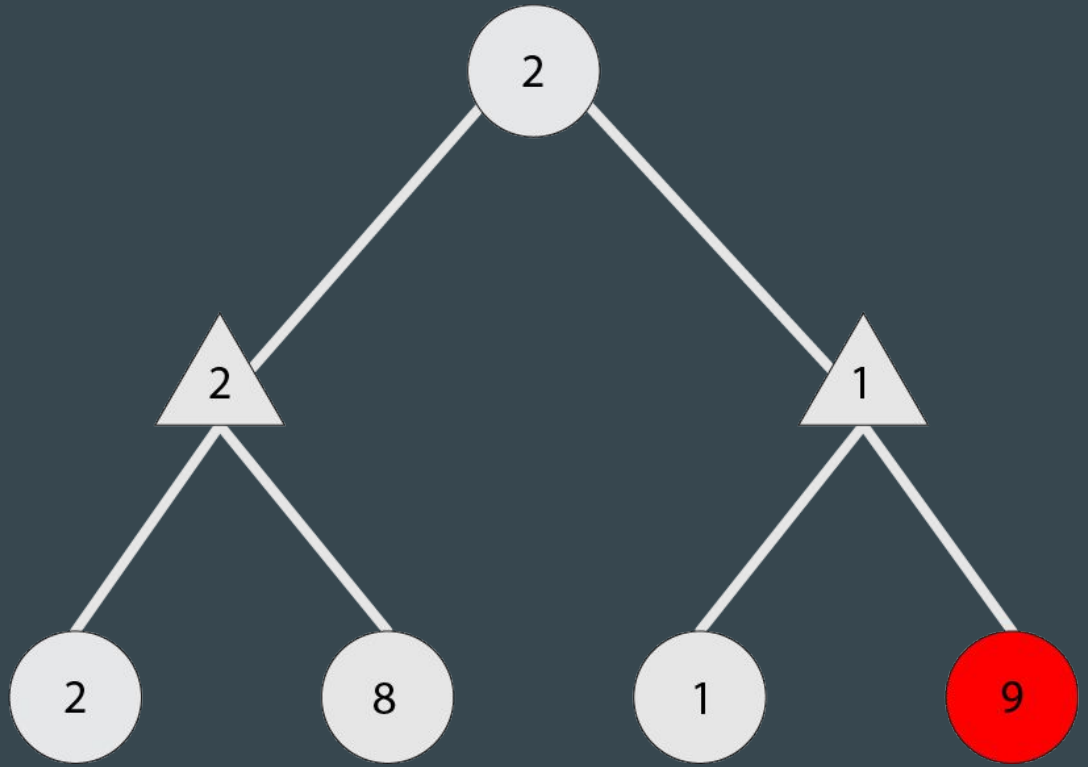
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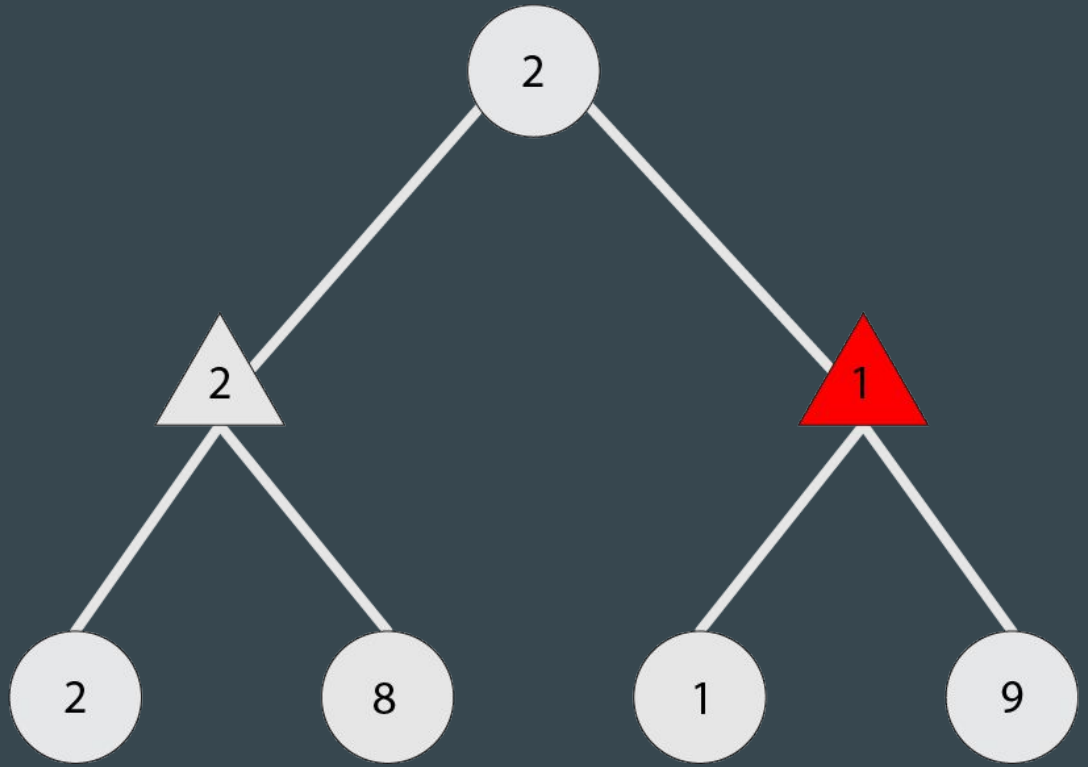
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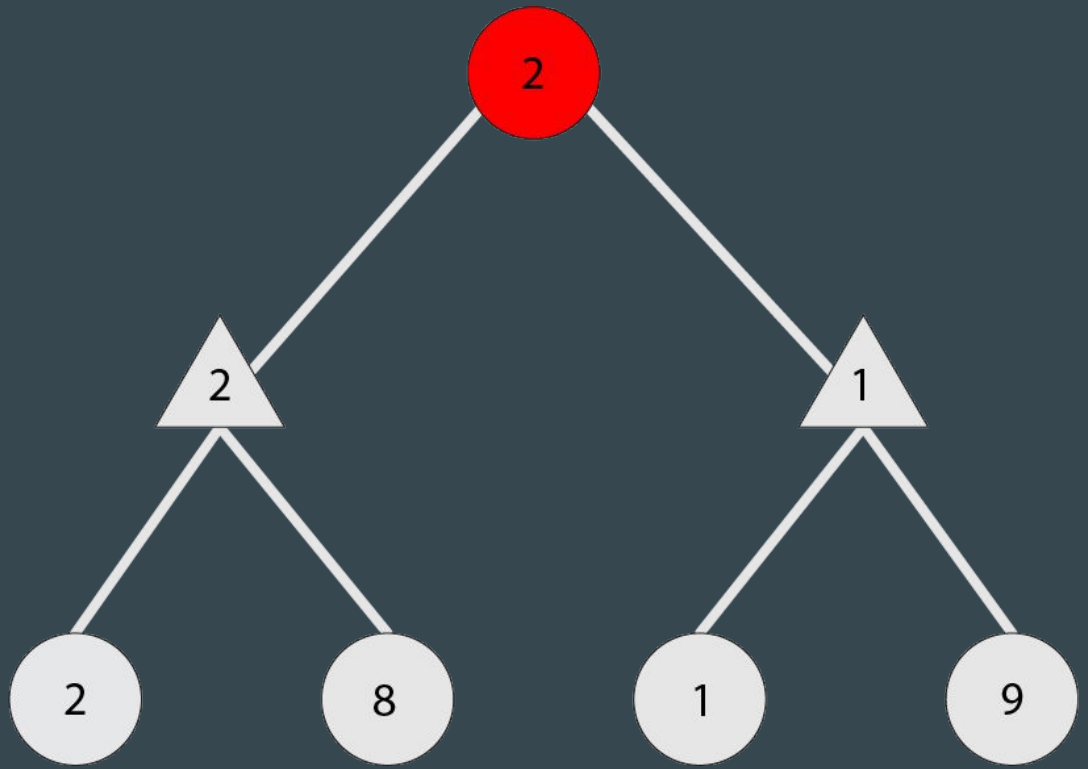
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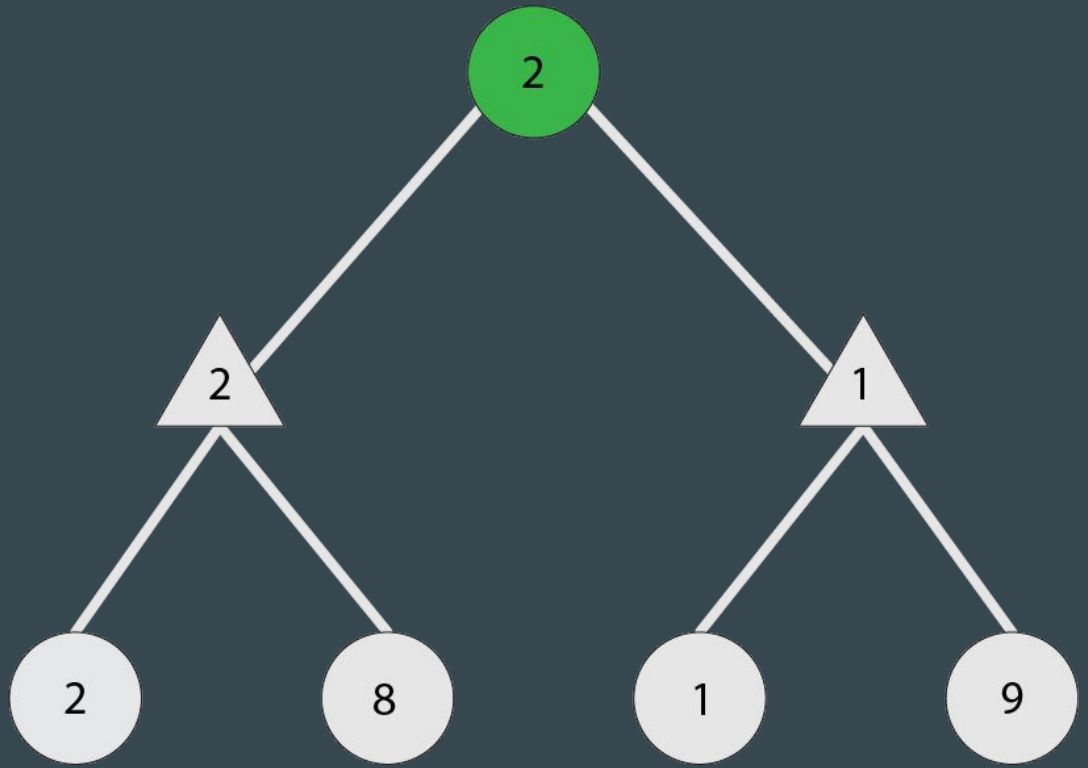
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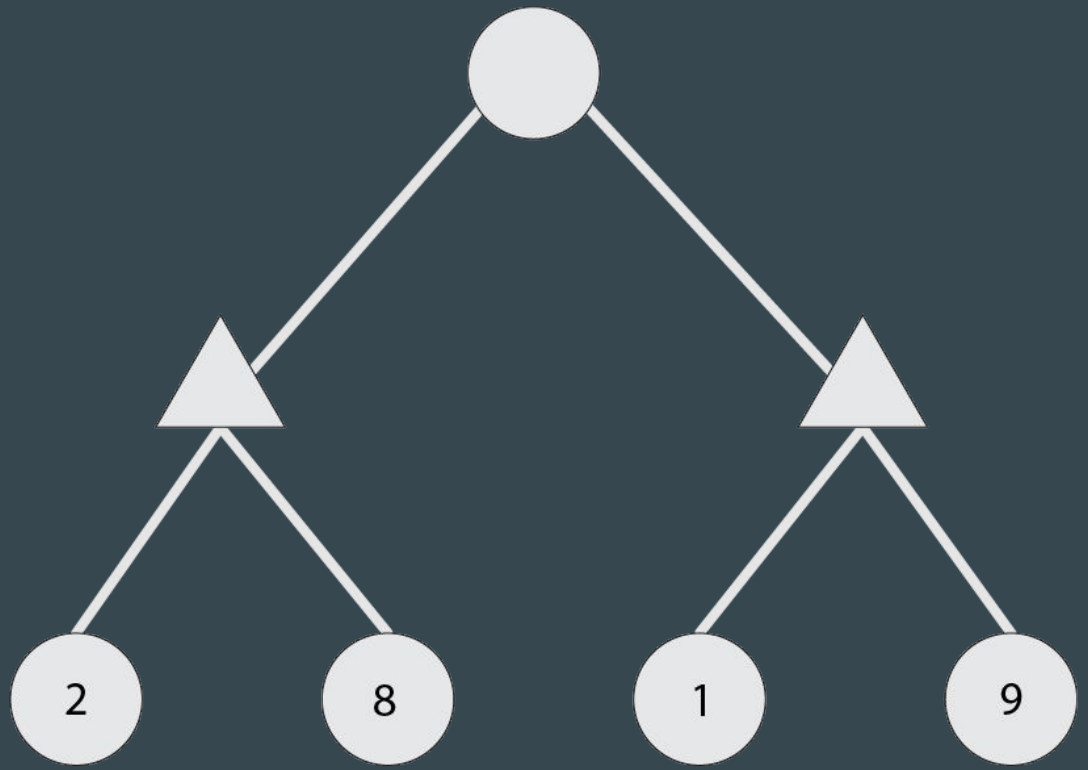
Alpha-Beta

Alpha-Beta

- Alpha-Beta is an improvement of minimax
- It allows for faster pruning
- Space complexity of $O(b^{m/2})$ in best case, $O(b^{m})$ in worst case
- Alpha value: best value for current player found so far
- Beta value: best value for opponent found so far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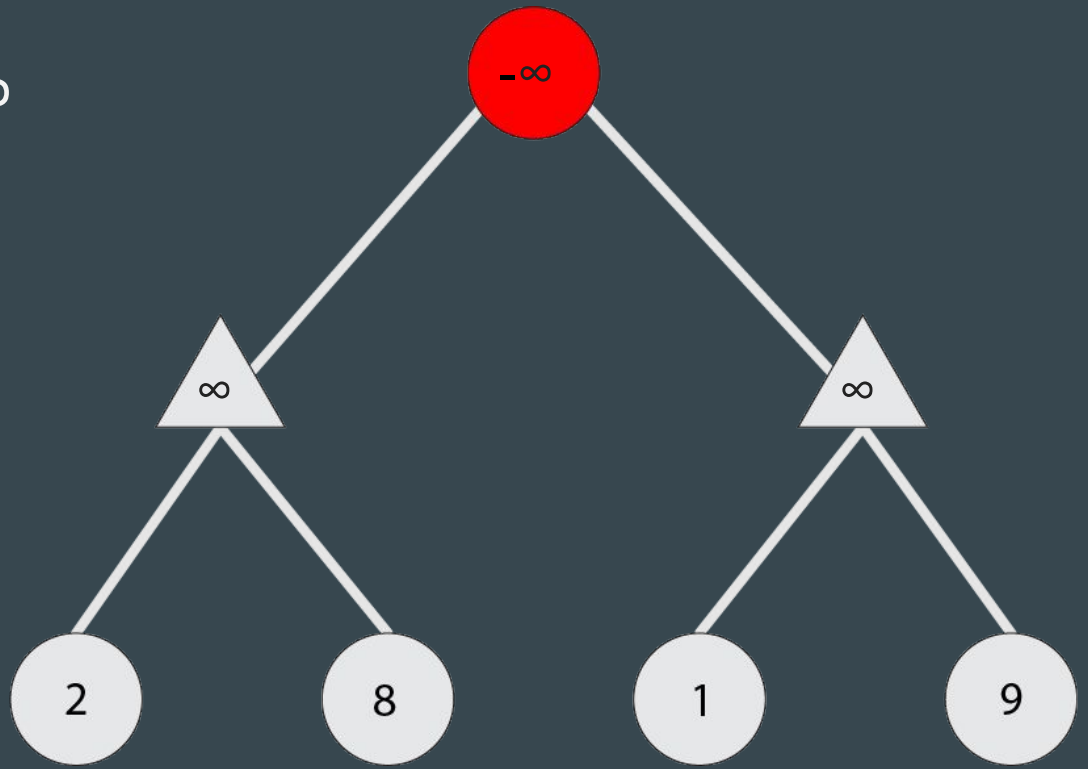
$\alpha = -\infty$

$\beta = \infty$



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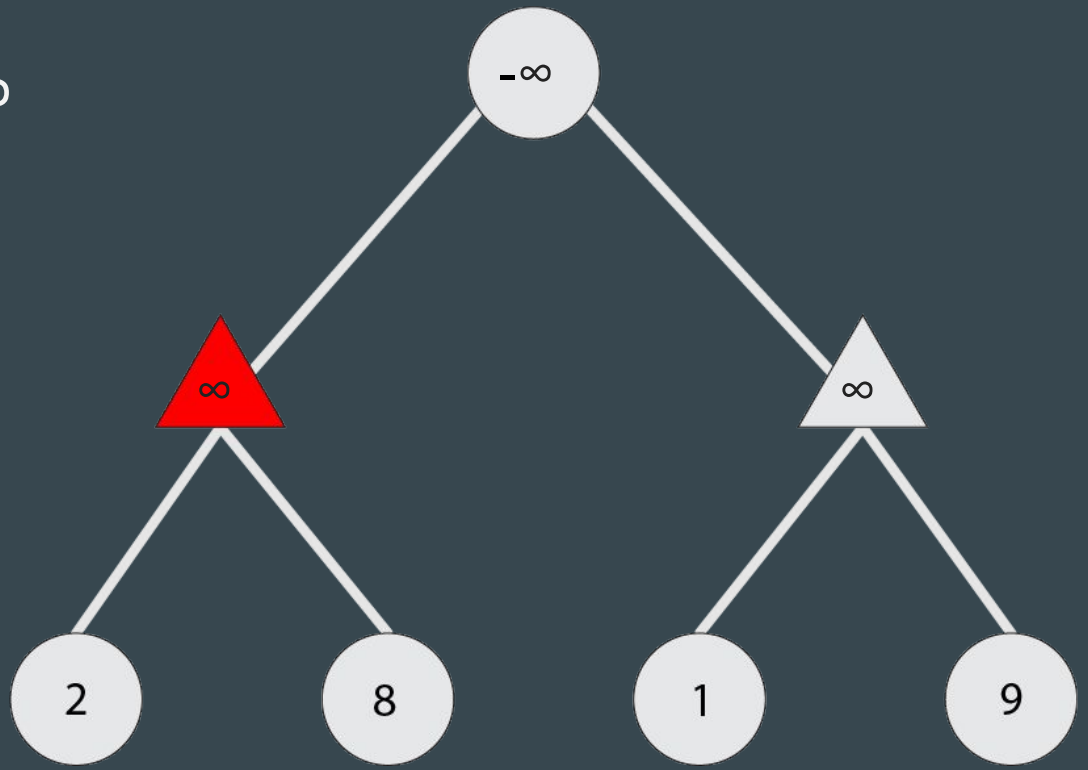
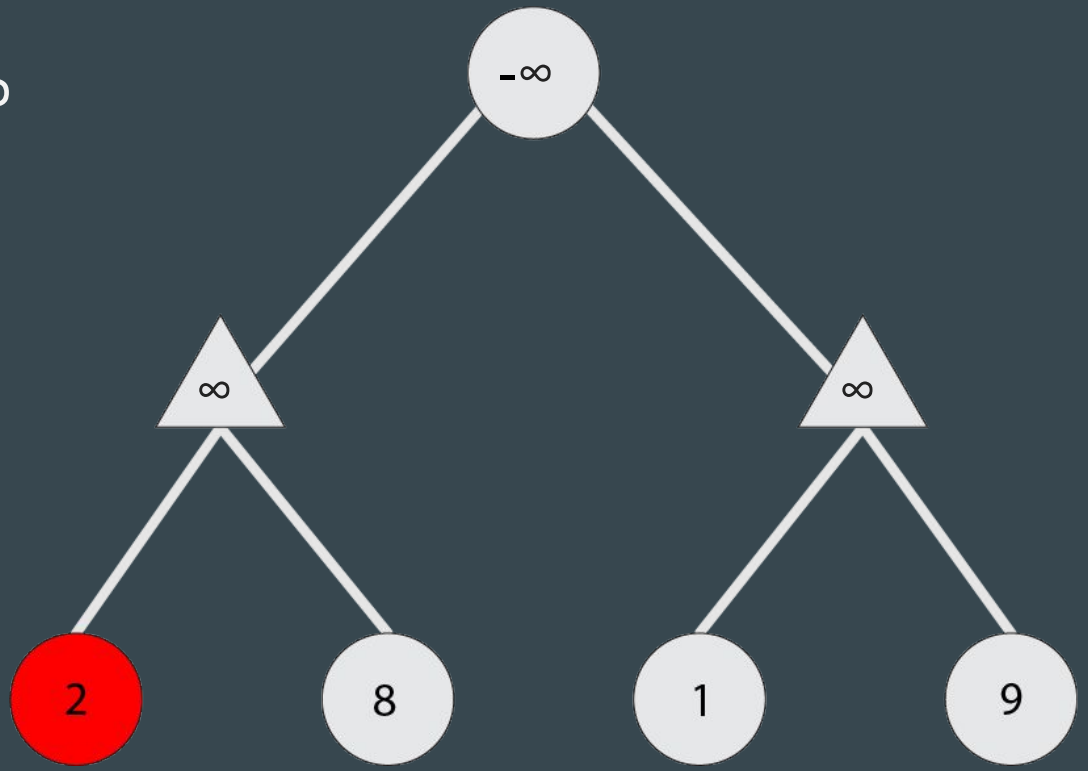


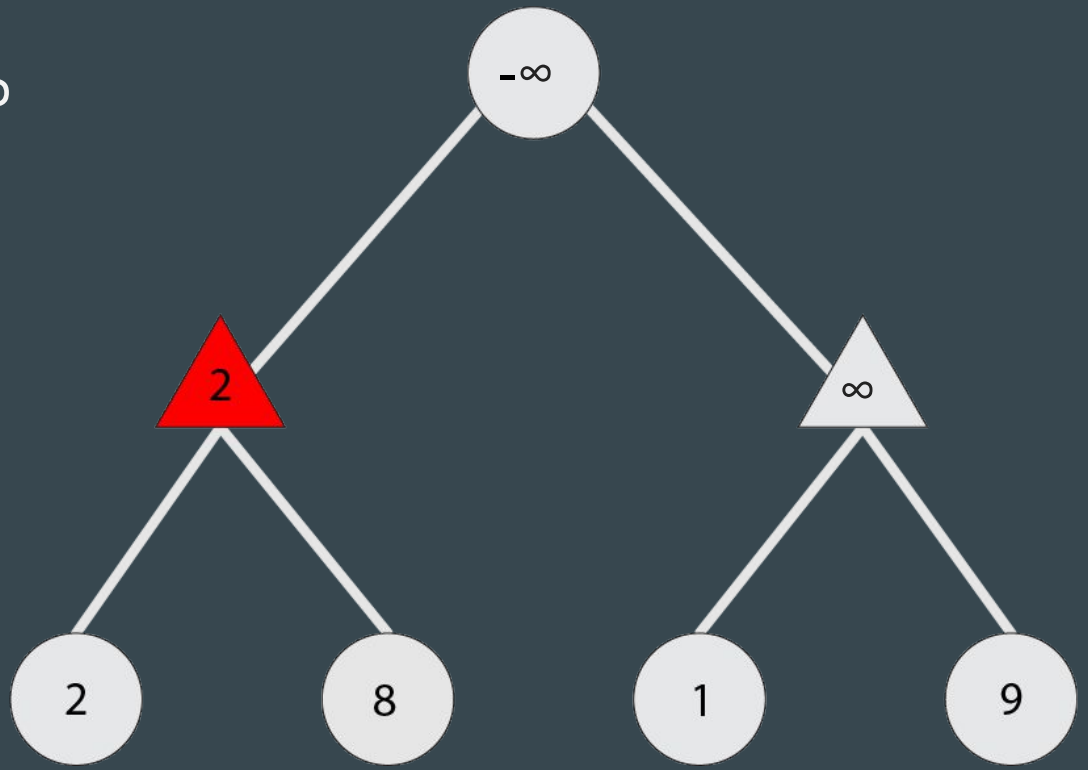
$\alpha = -\infty$

$\beta = \infty$



$\alpha = -\infty$

$\beta = 2$



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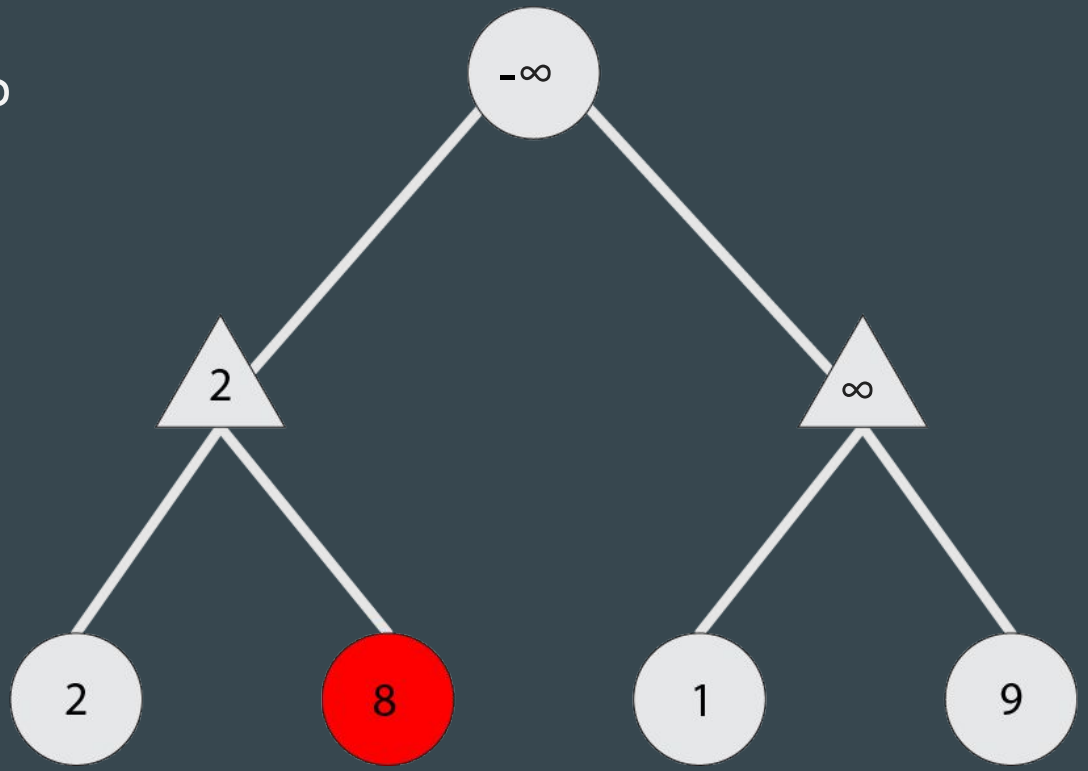

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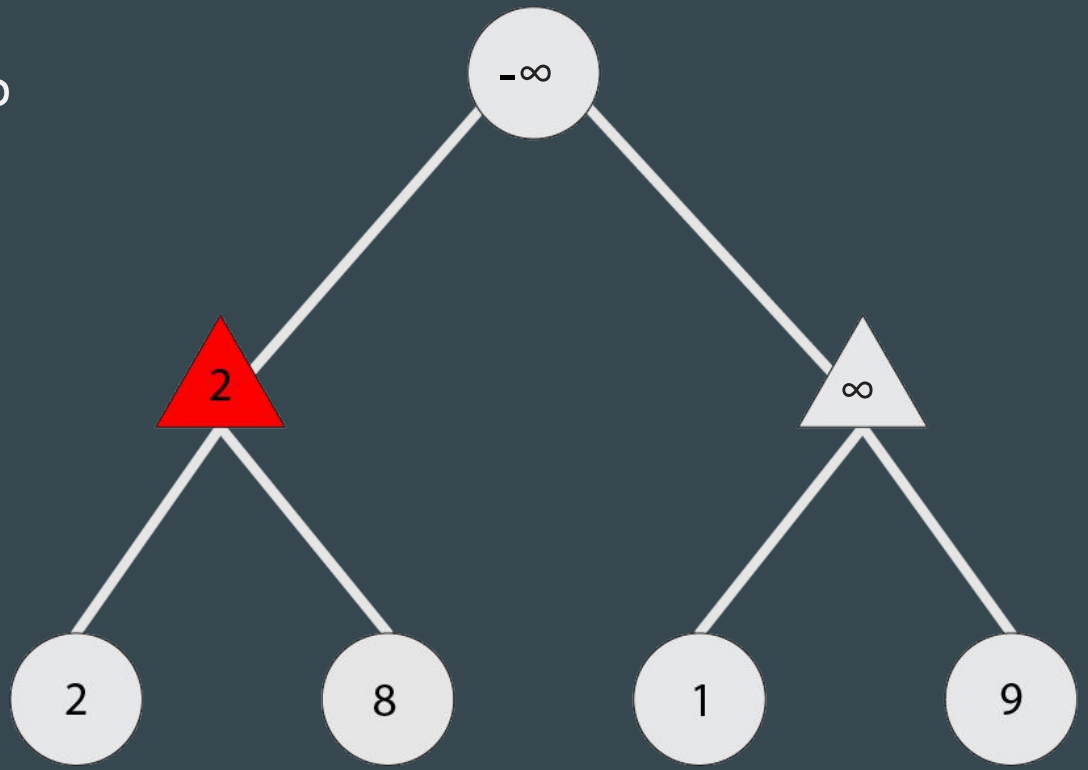
$\beta = 2$



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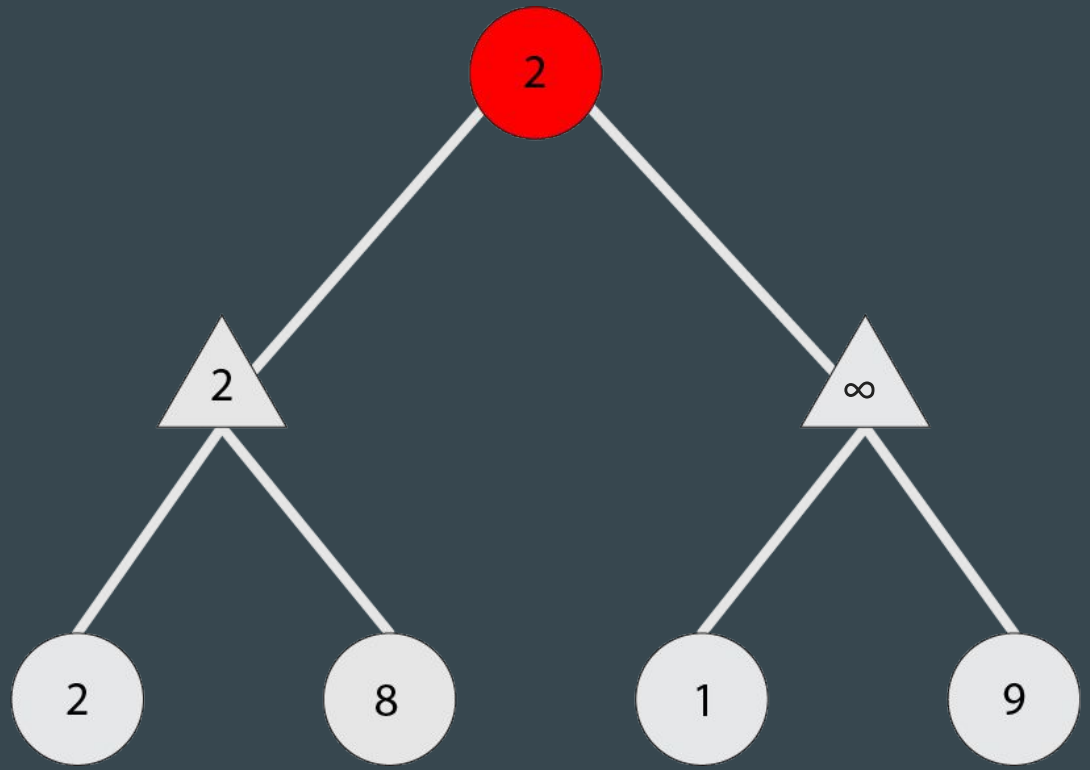

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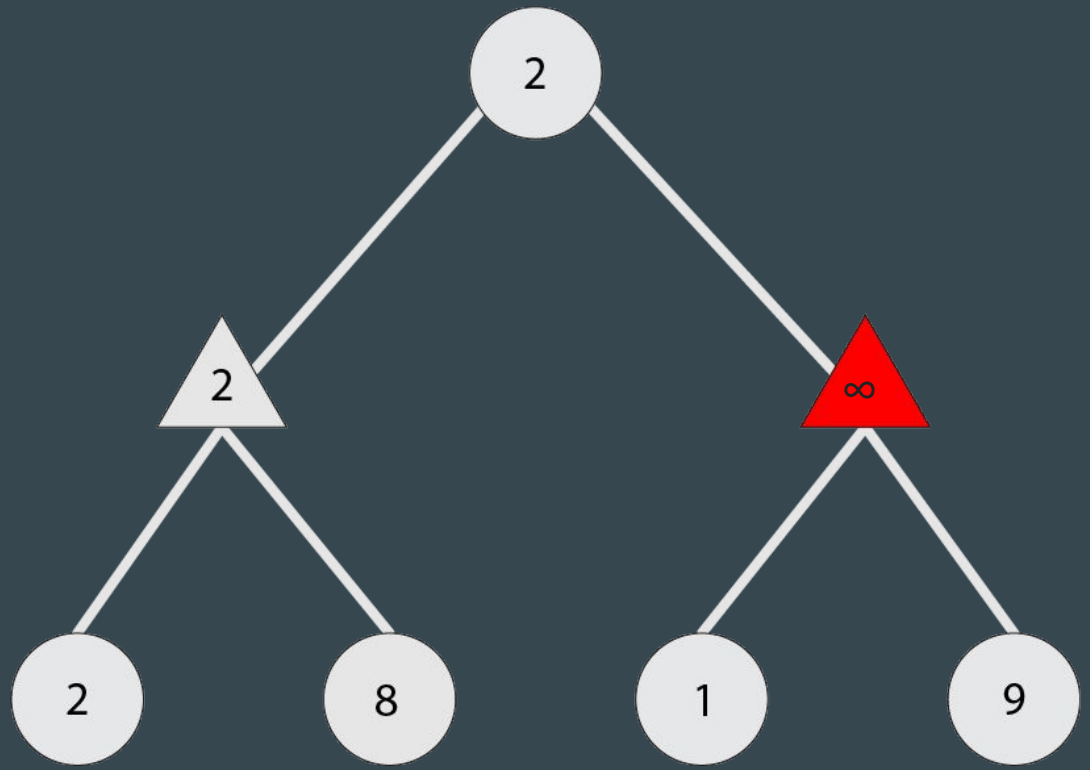

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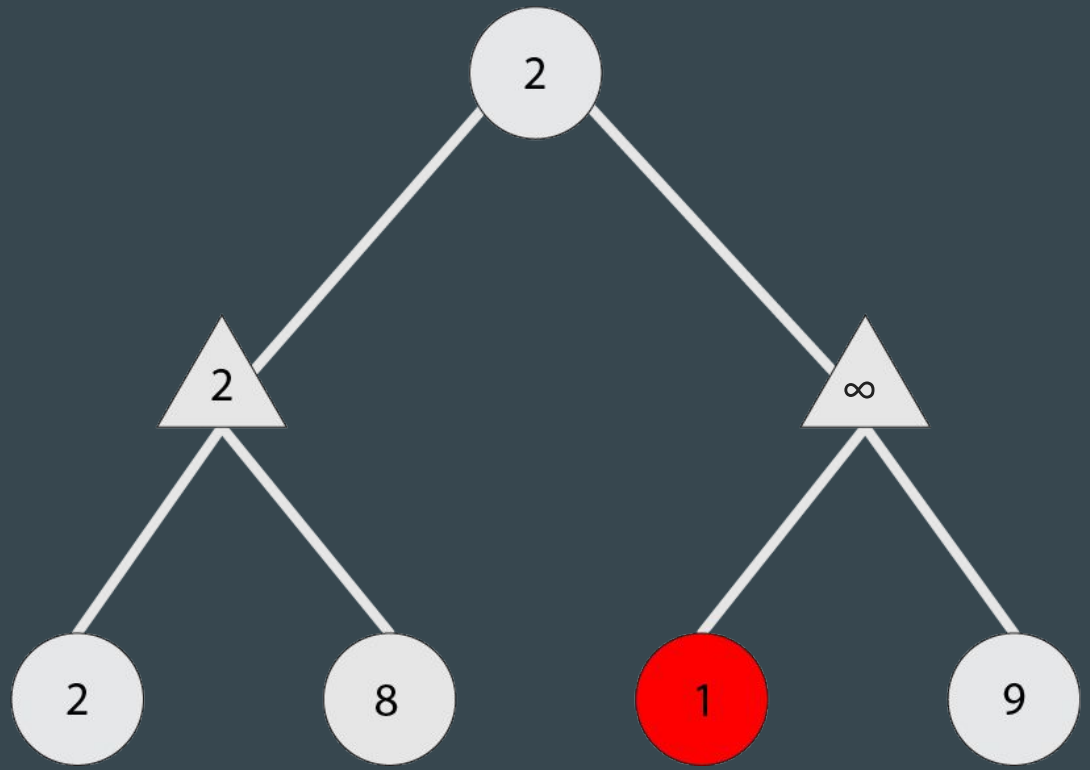

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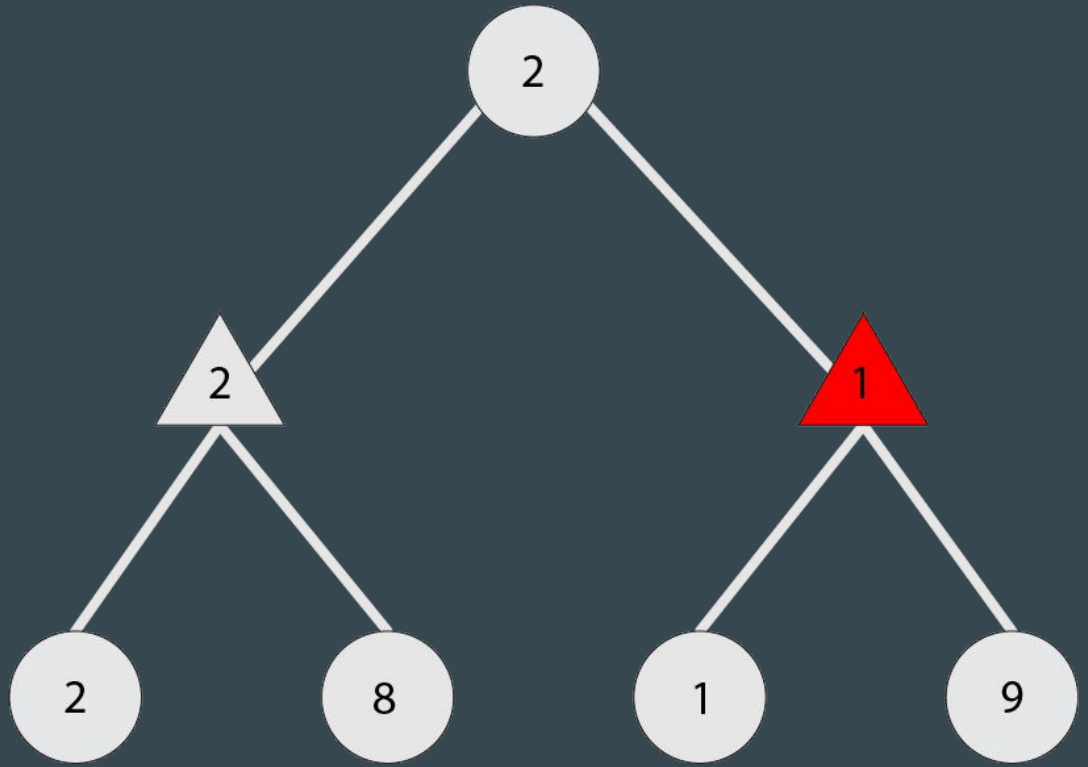

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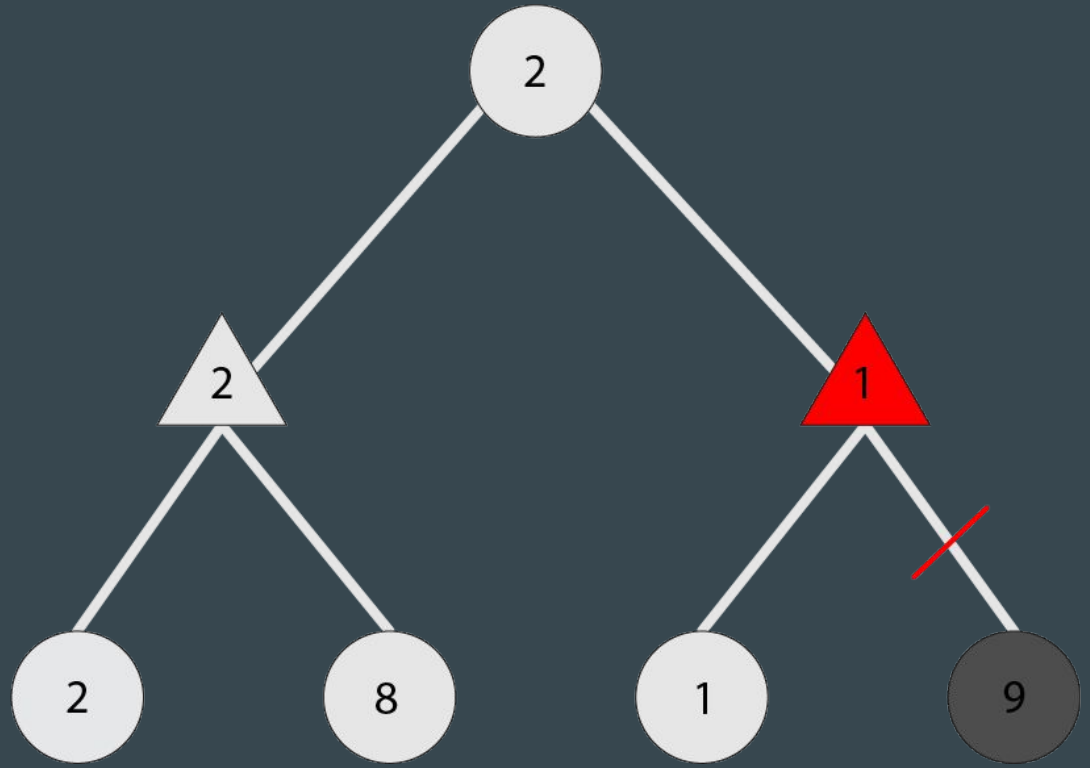

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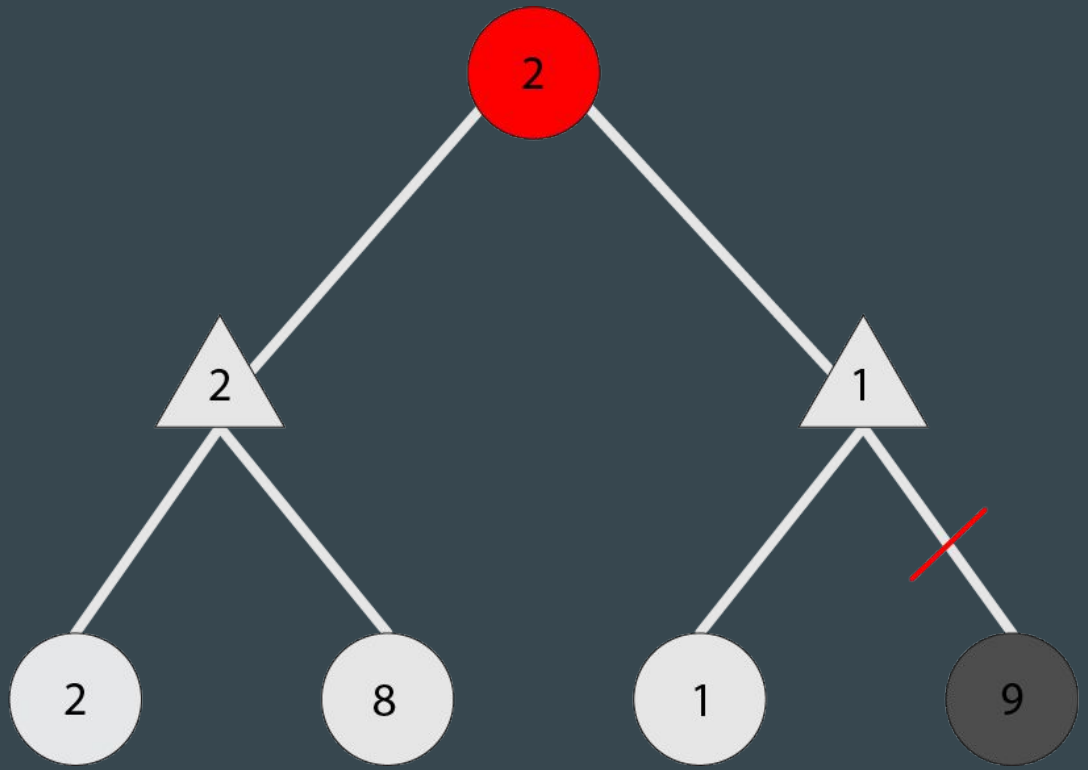

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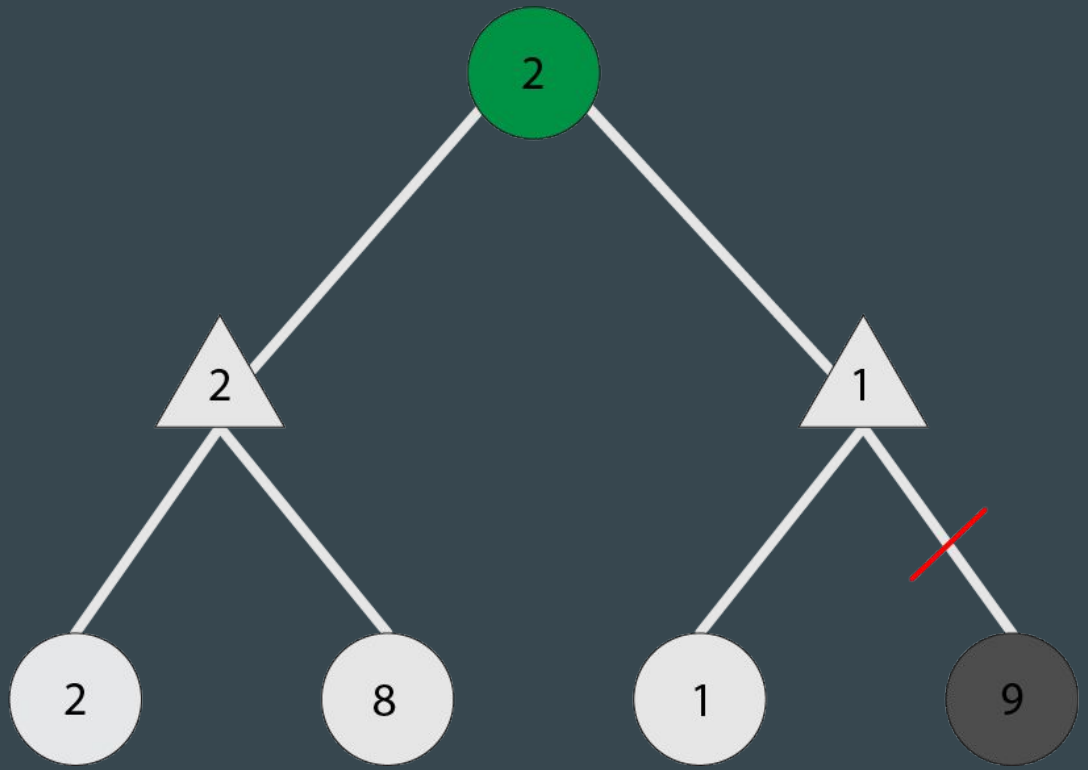

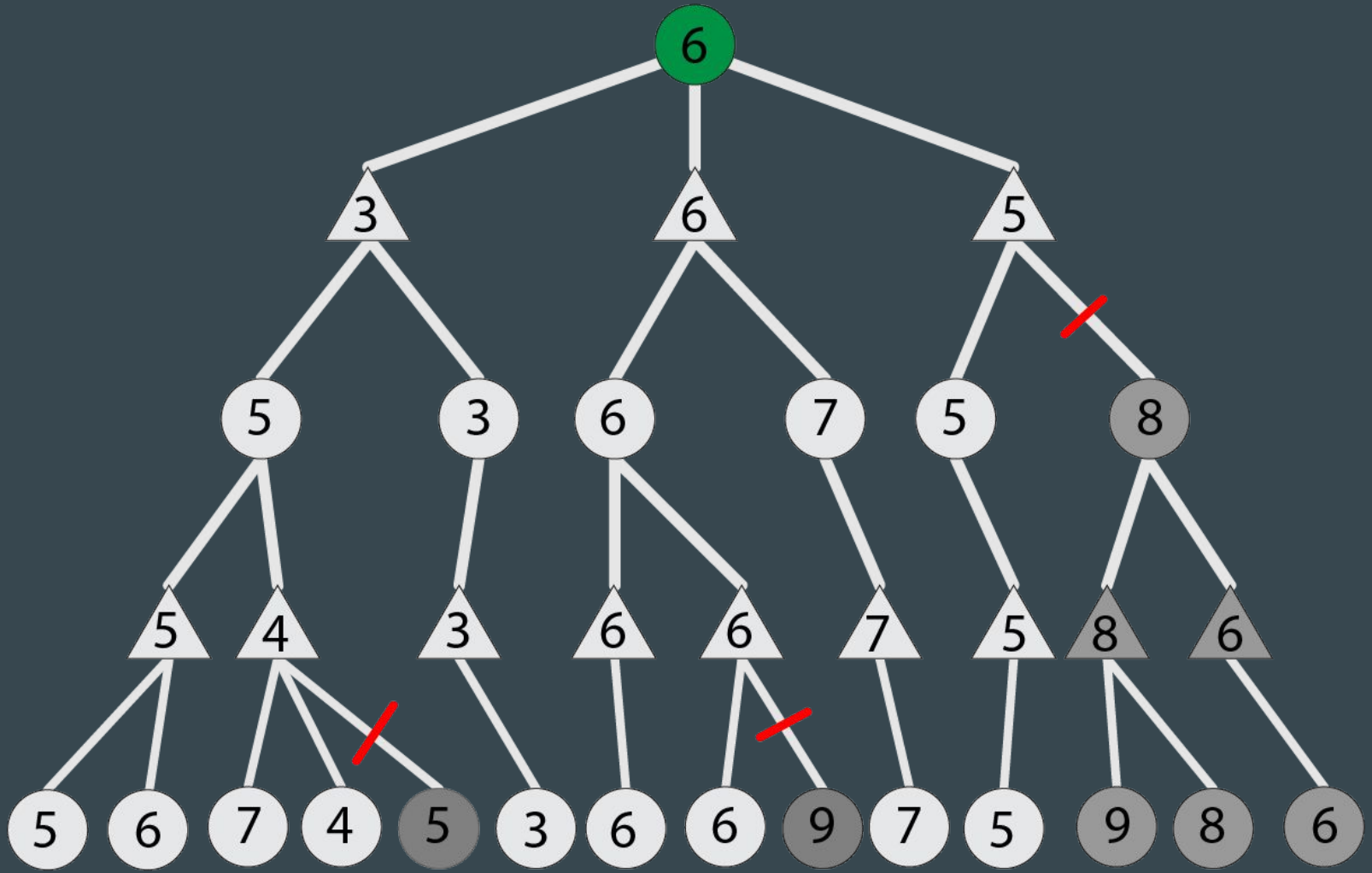
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Implementations in Chess Engines

What do we need to make a chess engine

- Board representation
- Evaluating position
- Alpha-Beta
- Enhancements
- Chess shortcuts and improvements

Board Representation

Piece Centric

- Piece-Lists
- Piece-Sets
- Bitboards

Board Centric

- Mailbox
- 8x8 Board
- 10x12 Board
- 0x88
- Vector Attacks

Board Representation

Piece Centric

- Piece-Lists

  Lists of the pieces on the board

Board Representation

Piece Centric

- Piece-Lists
- Piece-Sets

  One bit for each piece inside a
  32-bit word, similar to bitboards

Board Representation

Piece Centric

- Piece-Lists
- Piece-Sets
- Bitboards

  64 bit sets with one for each piece

Board Representation

Piece Centric

- Piece-Lists
- Piece-Sets
- Bitboards

Board Centric

- Mailbox

  Every square in a separately addressable memory element

Board Representation

Piece Centric

- Piece-Lists
- Piece-Sets
- Bitboards

Board Centric

- Mailbox
- 8x8 Board

  A two-dimensional array containing piece and empty square codes

Board Representation

Piece Centric

- Piece-Lists
- Piece-Sets
- Bitboards

Board Centric

- Mailbox
- 8x8 Board
- 10x12 Board

  Similar to 8x8 with more squares added to allow for knight jumps as moves

Board Representation

Piece Centric

- Piece-Lists
- Piece-Sets
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Board Centric

- Mailbox
- 8x8 Board
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  This uses one nibble to index the piece or empty square

Board Representation

Piece Centric

- Piece-Lists
- Piece-Sets
- Bitboards

Board Centric

- Mailbox
- 8x8 Board
- 10x12 Board
- 0x88

Evaluating Position

Historically uses an evaluation polynomial:

$$V = c_1 f_1 + c_2 f_2 + \ldots + c_n f_n$$

Recent advancements:

- Using Neural Networks for Evaluation *K Greer 2013*
- Better storage of board

Alpha-Beta

- This is the best search method found so far *K Greer 2013*
- How deep to go:
  - Deep blue beat Kasparov with a search depth of 14-16
  - Computers would need a search depth of 19 to consistently beat the current world champion
  - Modern computers search up to depth 35
- Move ordering, searching promising branches first

The impact of search depth on chess playing strength

Diogo R. Ferreira ICGA 2013

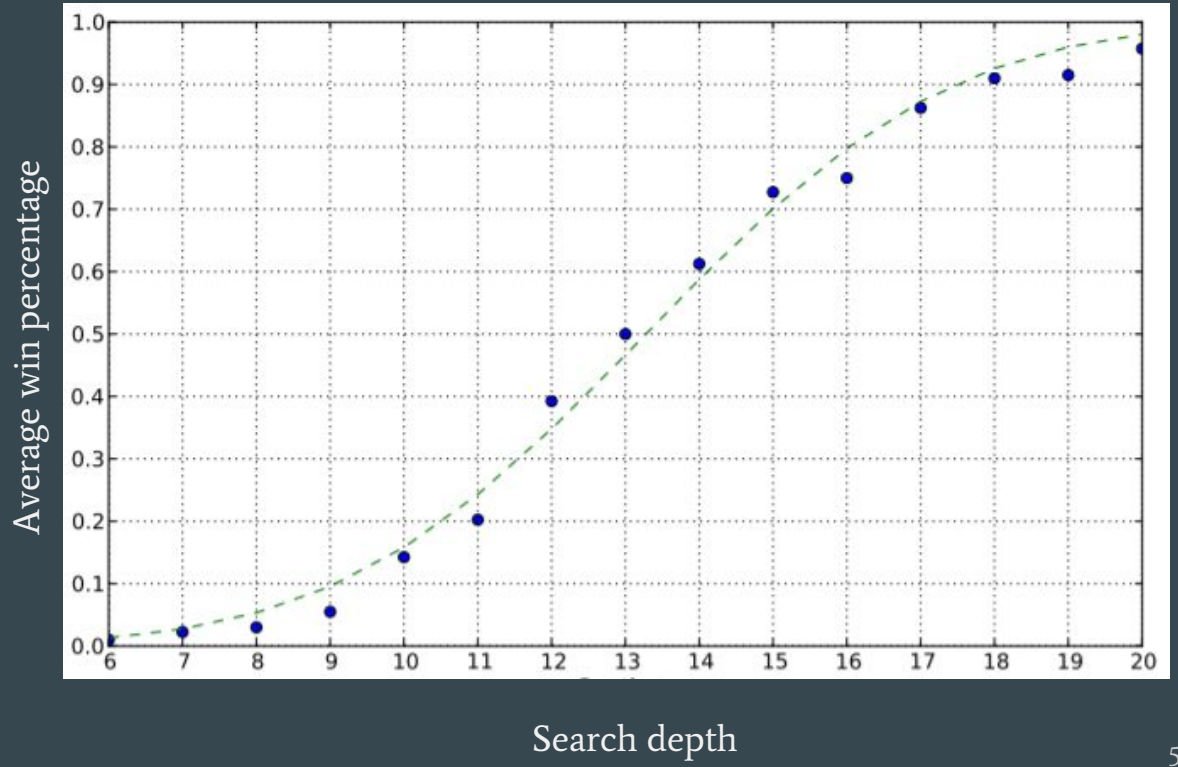
depth	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
6	0.50	0.29	0.15	0.07	0.02	0.03	0.01	0.01	0.00	0.01	0.00	0.01	0.00	0.00	0.00
7	0.71	0.50	0.30	0.16	0.07	0.04	0.01	0.02	0.01	0.00	0.00	0.00	0.01	0.00	0.00
8	0.84	0.70	0.50	0.24	0.12	0.06	0.04	0.03	0.01	0.01	0.01	0.00	0.01	0.01	0.01
9	0.93	0.84	0.76	0.50	0.28	0.18	0.10	0.06	0.04	0.03	0.01	0.01	0.01	0.00	0.00
10	0.98	0.93	0.89	0.72	0.50	0.32	0.26	0.14	0.08	0.06	0.03	0.03	0.01	0.01	0.01
11	0.97	0.96	0.94	0.81	0.68	0.50	0.34	0.20	0.15	0.11	0.09	0.06	0.03	0.03	0.03
12	0.99	0.98	0.96	0.90	0.74	0.66	0.50	0.39	0.27	0.19	0.12	0.08	0.04	0.04	0.04
13	0.99	0.98	0.97	0.94	0.86	0.80	0.61	0.50	0.39	0.27	0.25	0.14	0.09	0.09	0.04
14	1.00	0.99	0.99	0.95	0.92	0.85	0.73	0.61	0.50	0.38	0.30	0.21	0.16	0.15	0.07
15	0.99	1.00	0.99	0.97	0.94	0.89	0.81	0.73	0.62	0.50	0.40	0.33	0.24	0.21	0.15
16	1.00	1.00	0.99	0.98	0.97	0.91	0.89	0.75	0.70	0.60	0.50	0.40	0.31	0.27	0.15
17	0.99	1.00	1.00	0.99	0.97	0.94	0.92	0.86	0.79	0.67	0.60	0.50	0.44	0.32	0.26
18	1.00	0.99	0.99	0.99	0.99	0.97	0.95	0.91	0.84	0.76	0.69	0.56	0.50	0.45	0.34
19	1.00	1.00	0.99	1.00	0.99	0.97	0.96	0.92	0.85	0.79	0.73	0.68	0.55	0.50	0.43
20	1.00	1.00	0.99	1.00	0.99	0.97	0.96	0.96	0.93	0.85	0.85	0.74	0.66	0.57	0.50

Average score over 200 games irrespective of color

The impact of search depth on chess playing strength

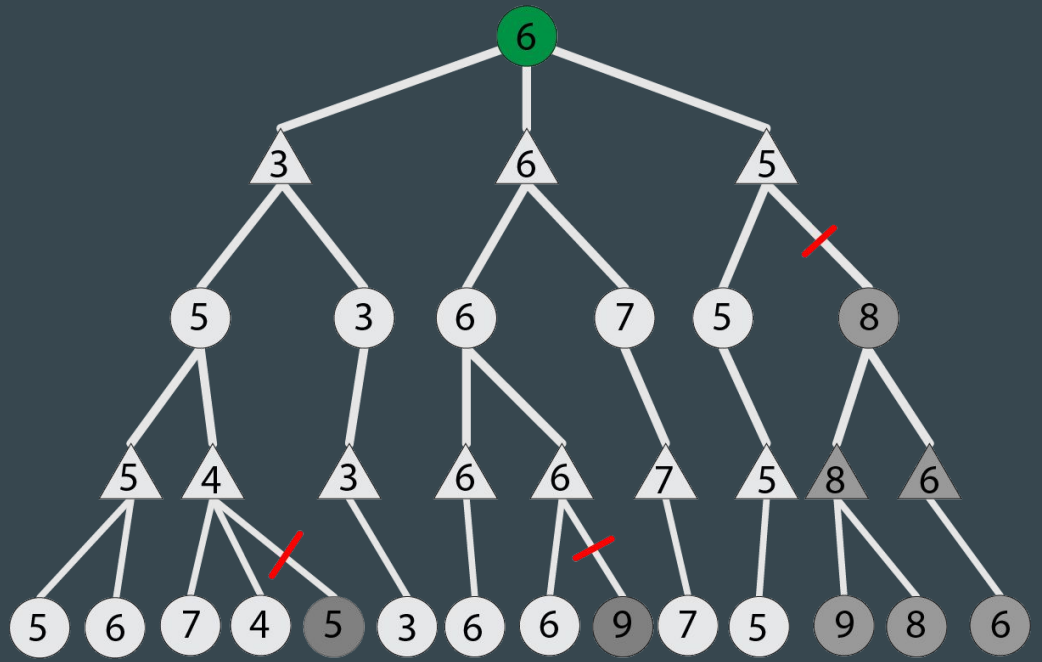
Diogo R. Ferreira ICGA 2013

- Each move added or removed significantly changes engine strength
- Played using HOUDINI 1.5a 64-bit
- A search depth of 20 is rated 2893.9, top human is 2838



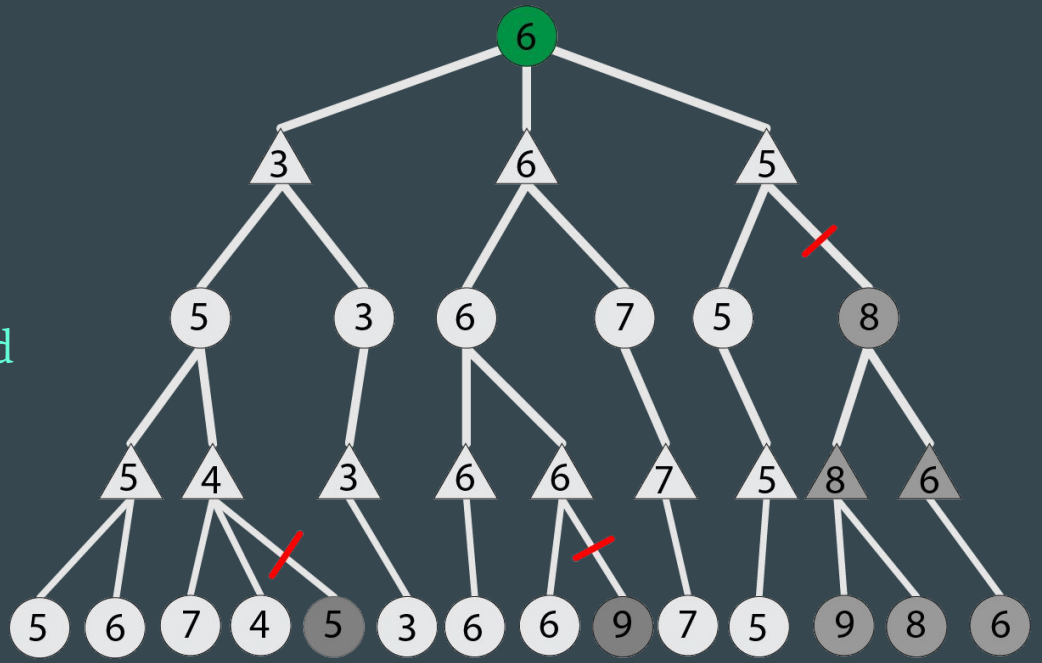
Enhancements

- Limiting search space
  - Starting with more promising branches and going deeper in these



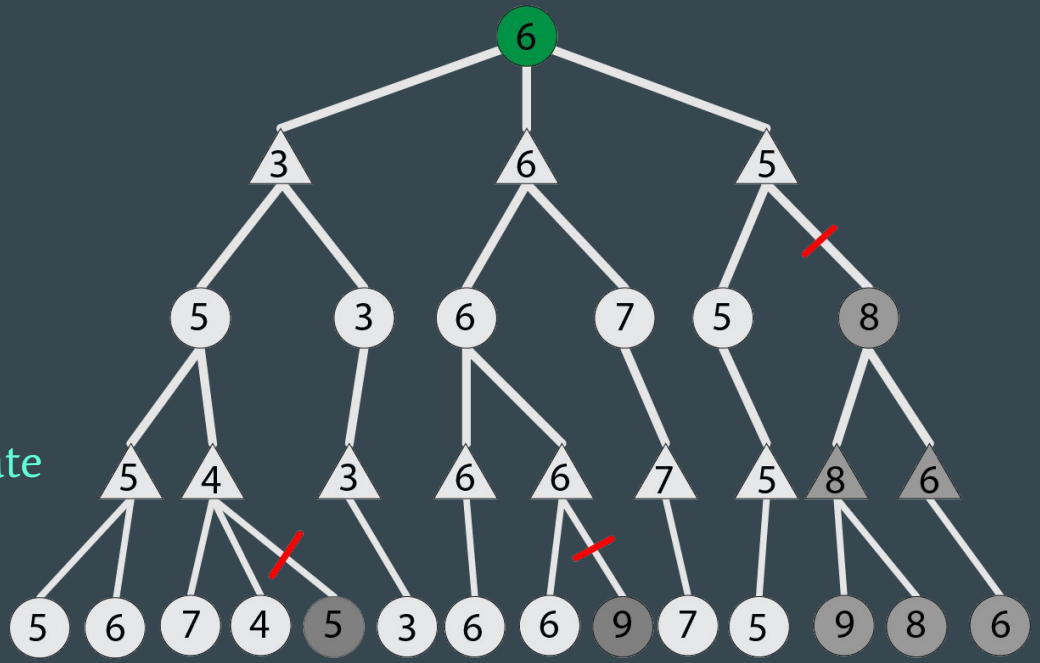
Enhancements

- Limiting search space
- Iterative deepening
  - Starting the search shallow and deepening



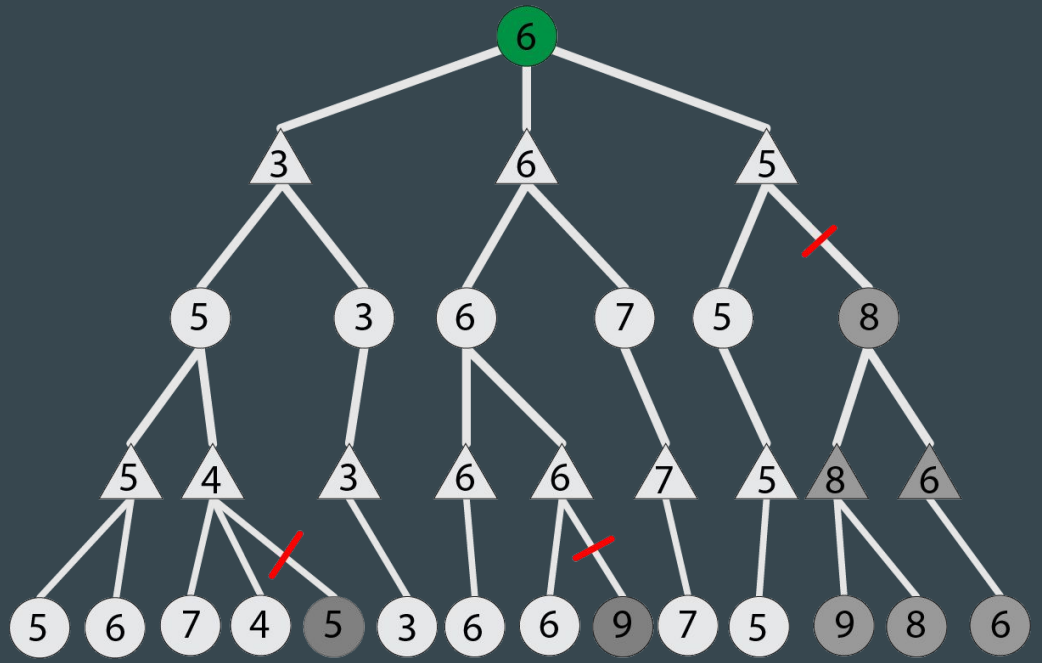
Enhancements

- Limiting search space
- Iterative deepening
- Parallel computing
  - Giving each branch to a separate core to speed up the search



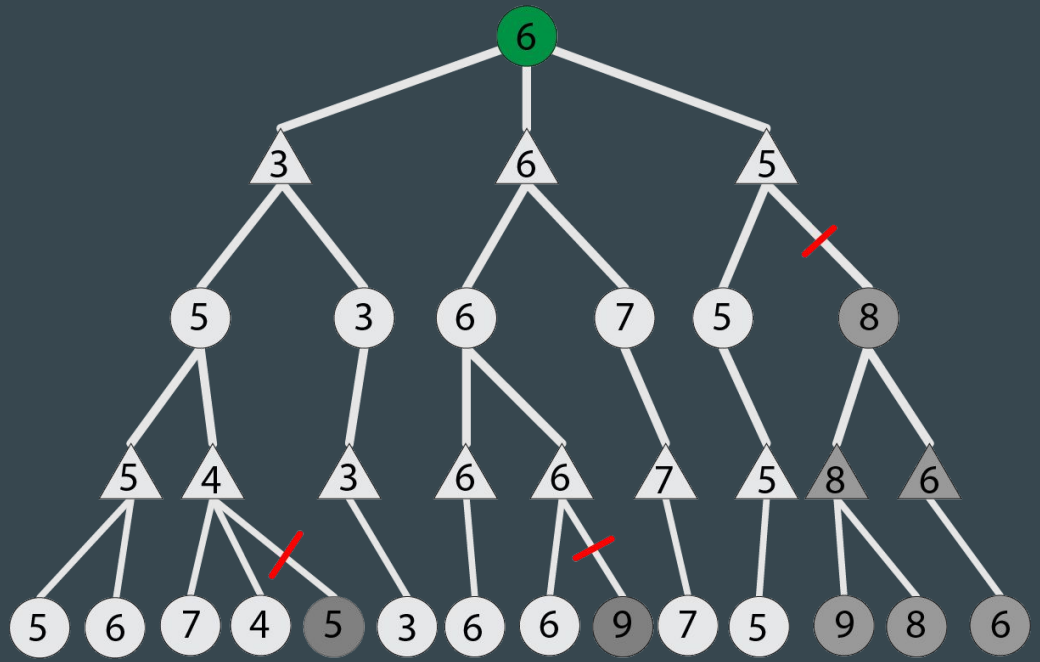
Enhancements

- Limiting search space
- Iterative deepening
- Parallel computing
- Uneven tree development
  - Chess is not the same depth every option



Enhancements

- Limiting search space
- Iterative deepening
- Parallel computing
- Uneven tree development



Chess Shortcuts and Improvements

- 3 parts of the game:
  - Opening
  - Middlegame
  - Endgame
- Principal Variation
  - A sequence of moves that programs consider best and expect to be played

Opening

Middle Game

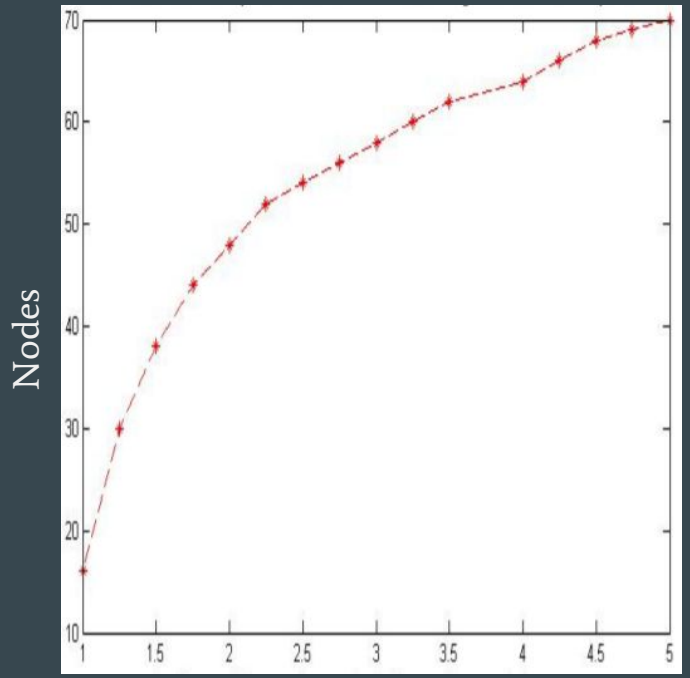
End Game

Information and search in computer chess

Alexandru Godescu, ETH Zurich 2011

- Humans look at certain pieces first to find the best moves
- Using information theory we can choose better nodes and depth to search
- These two things are proven to have very similar results

Decrease of importance of information moves vs plies



Reduction of importance to information gaining moves

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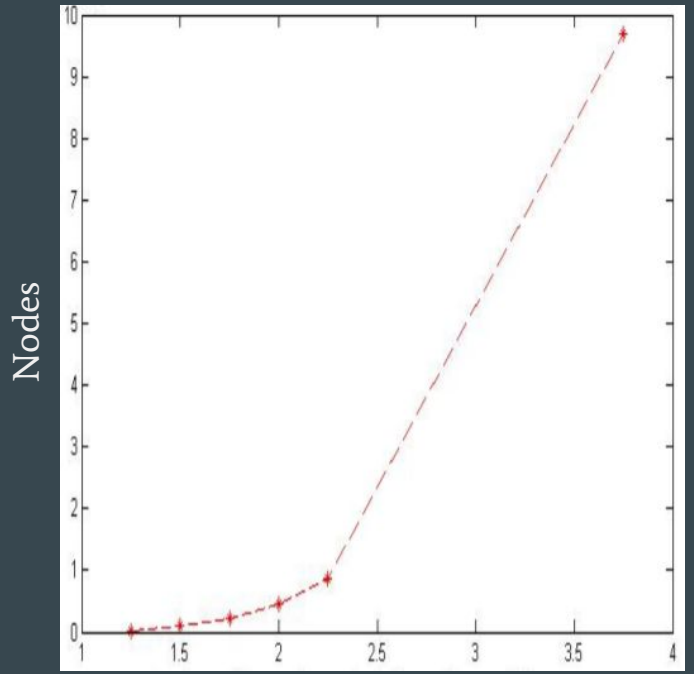
Decrease of importance of information moves vs nodes increase



Reduction of importance to information gaining moves

Conclusion

- Current chess engines are beyond human players
- Recent advancements in Engines relate largely to improving evaluation of the board
- Chess Engines use more than Alpha-Beta
- Alpha-Beta is the most effective method for chess and the one used in the engines

Questions?

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