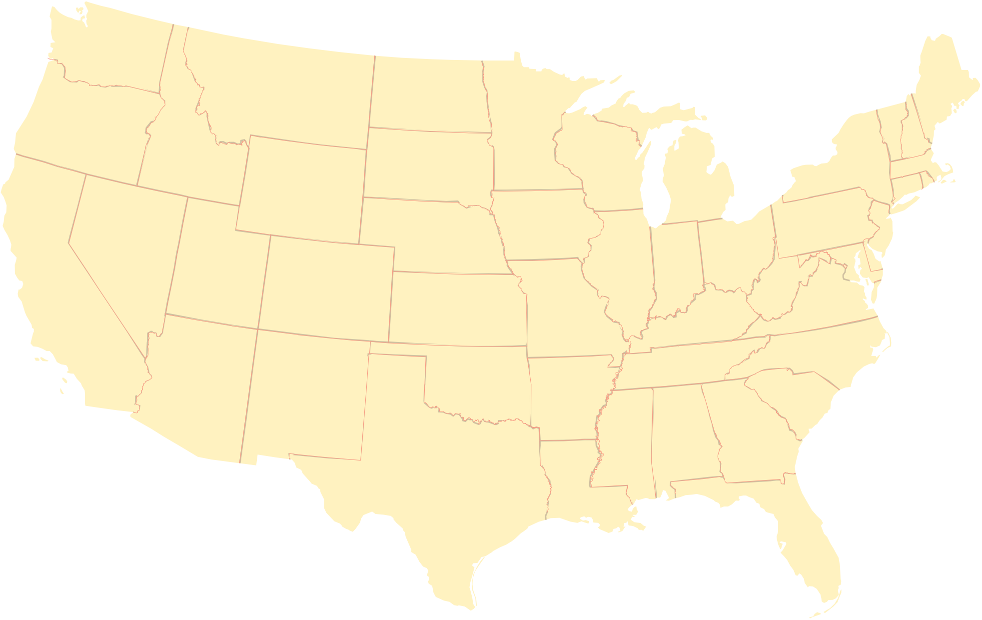
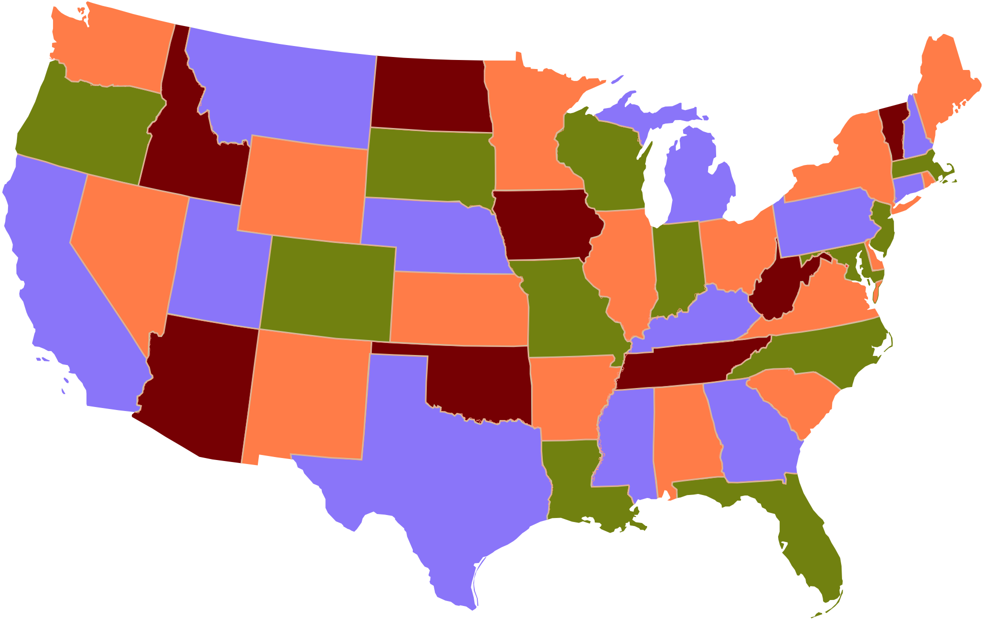
Conflict-Free Vertex Coloring of Planar Graphs

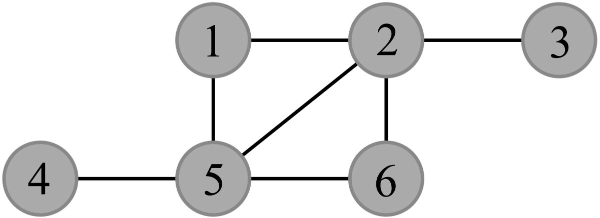
Shawn Seymour

April 15, 2017



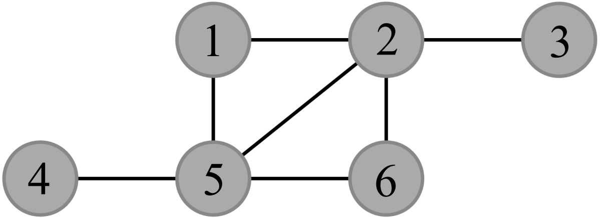


Overview

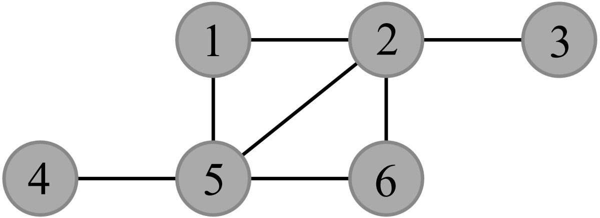


- **Simple graph**: undirected graph with no loops.

Graph Theory

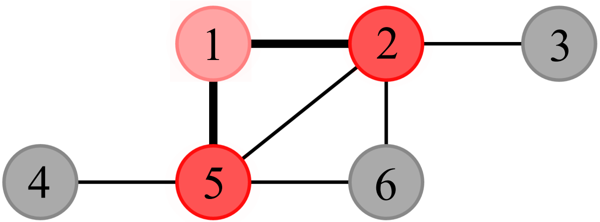


- **Simple graph**: undirected graph with no loops.
- **Planar graph**: no edges cross.



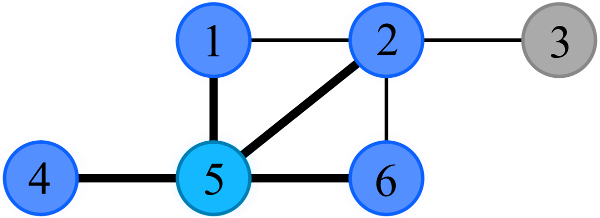
- **Neighborhood** of a vertex: a set of all adjacent vertices and the vertex itself.

Graph Theory



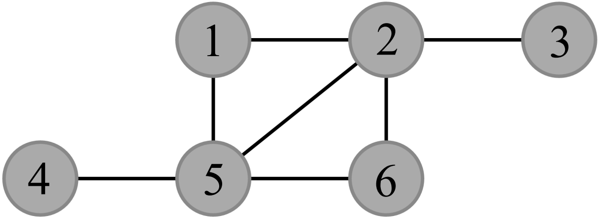
- **Neighborhood** of a vertex: a set of all adjacent vertices and the vertex itself.
- Example: The neighborhood of vertex 1: $\{1, 2, 5\}$.

Graph Theory



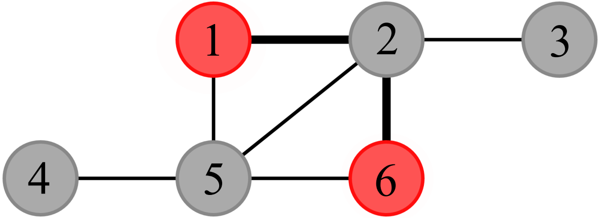
- **Neighborhood** of a vertex: a set of all adjacent vertices and the vertex itself.
- Example: The neighborhood of vertex 5: $\{1, 2, 4, 5, 6\}$.

Graph Theory

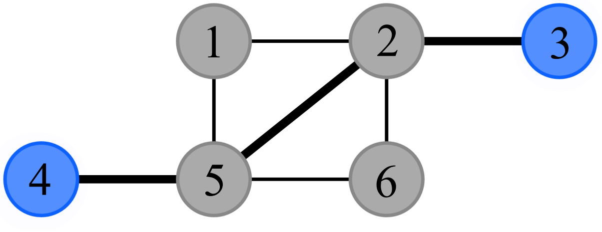


- **Distance**: smallest number of edges to get from one vertex to another.

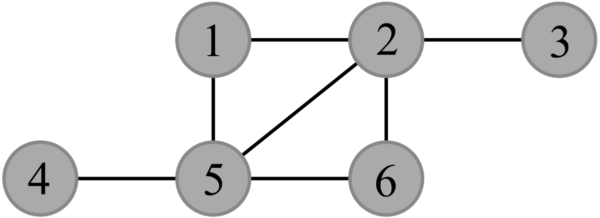
Graph Theory



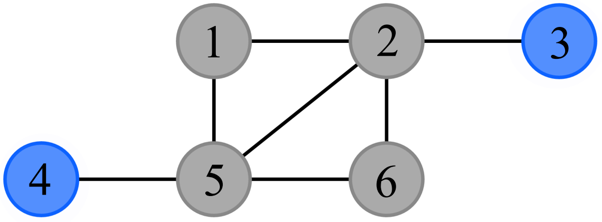
- **Distance**: smallest number of edges to get from one vertex to another.
- Example: distance from vertex 1 to 6: **2**.



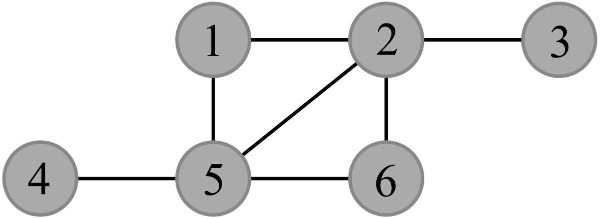
- **Distance**: smallest number of edges to get from one vertex to another.
- Example: distance from vertex 3 to 4: **3**.



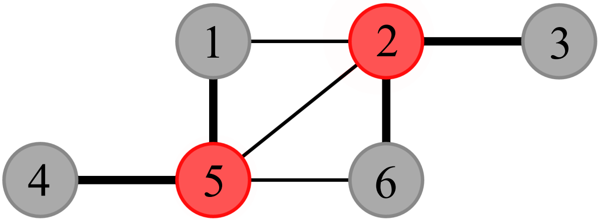
- **Distance-3-set**: contains all vertices with exactly distance 3 from each other.



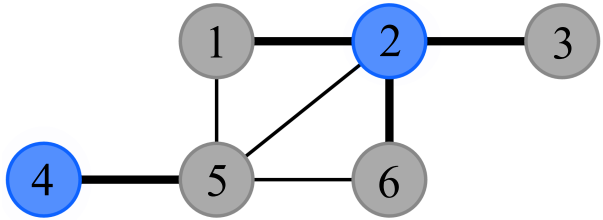
- **Distance-3-set**: contains all vertices with exactly distance 3 from each other.
- Example: The only possible distance-3-set: $\{3, 4\}$.



- **Dominating set**: all vertices not in the set must have distance 1 to some vertex within the set.



- **Dominating set**: all vertices not in the set must have distance 1 to some vertex within the set.
- Example: $\{2, 5\}$



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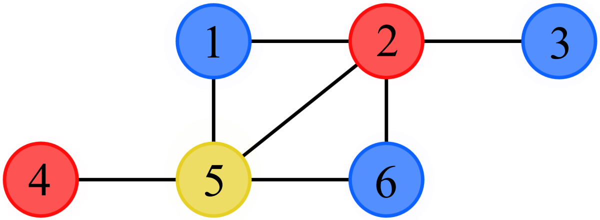
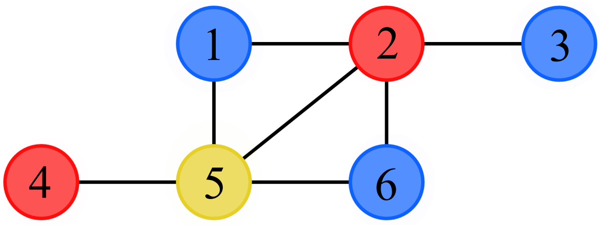
- Example: $\{2, 4\}$

Vertex Coloring



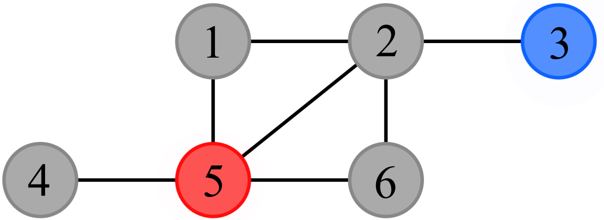
- A **proper vertex coloring** assigns colors to *every* vertex such that no two adjacent vertices share the same color.

Vertex Coloring



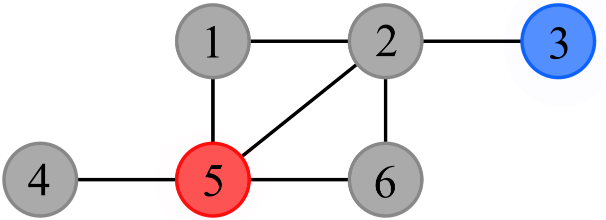
- A **proper vertex coloring** assigns colors to *every* vertex such that no two adjacent vertices share the same color.
- The **chromatic number** is the minimum number of colors needed to properly color a graph.

Conflict-Free Coloring



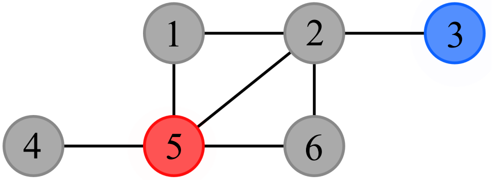
- A **conflict-free coloring** assigns colors to *some* vertices such that the neighborhood of every vertex contains at least one uniquely colored vertex.

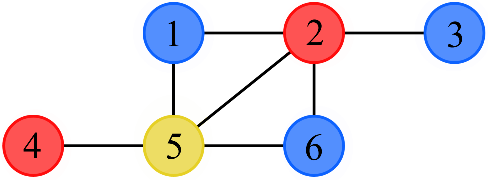
Conflict-Free Coloring



- A **conflict-free coloring** assigns colors to *some* vertices such that the neighborhood of every vertex contains at least one uniquely colored vertex.
- Proper vertex colorings are also conflict-free colorings.

Conflict-Free Coloring





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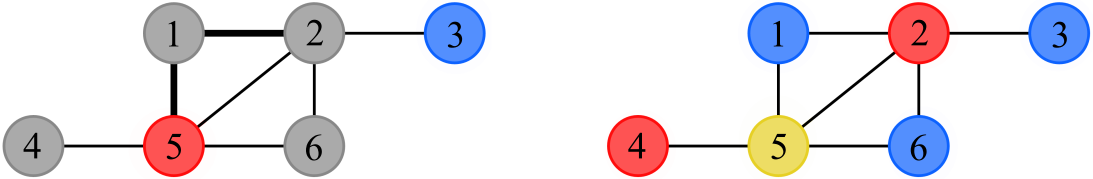
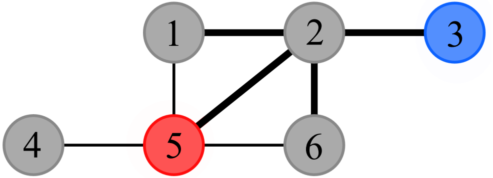
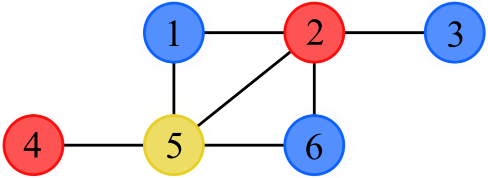


1: **red**

Conflict-Free Coloring

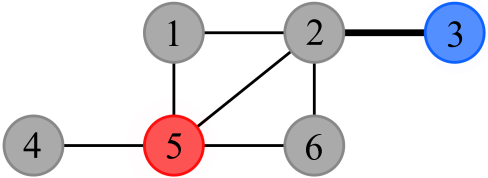


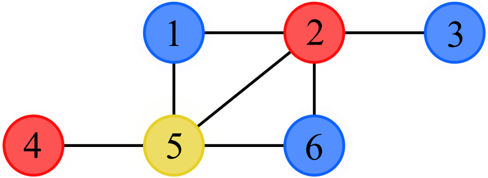


1: **red**

2: **blue**

Conflict-Free Coloring



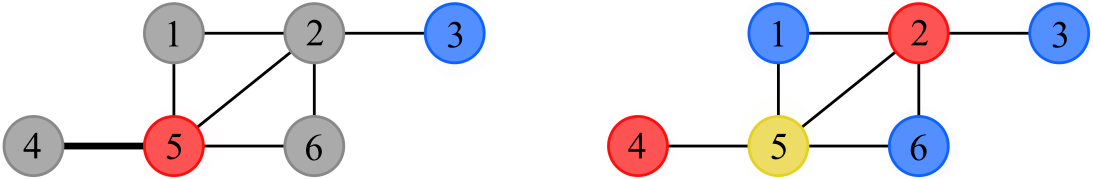


1: **red**

2: **blue**

3: **blue**

Conflict-Free Coloring



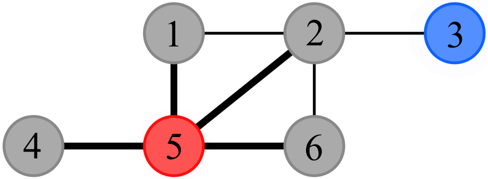
1: **red**

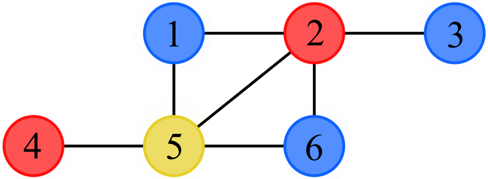
2: **blue**

3: **blue**

4: **red**

Conflict-Free Coloring





1: **red**

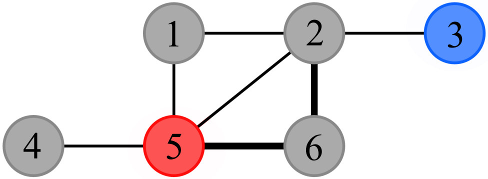
2: **blue**

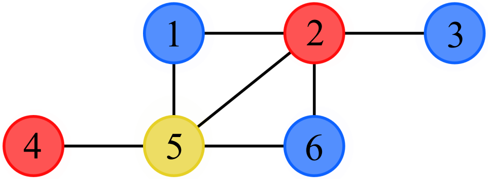
3: **blue**

4: **red**

5: **red**

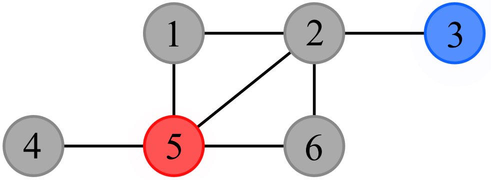
Conflict-Free Coloring

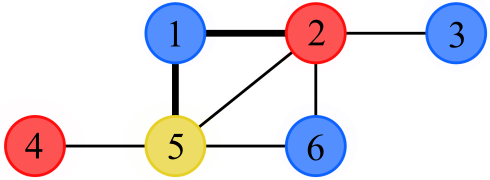




1: **red**	4: **red**
2: **blue**	5: **red**
3: **blue**	6: **red**

Conflict-Free Coloring





1: **red**

2: **blue**

3: **blue**

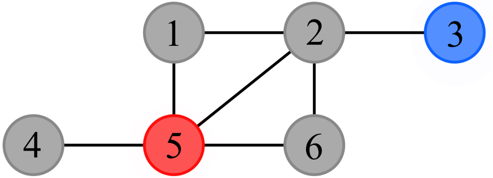
4: **red**

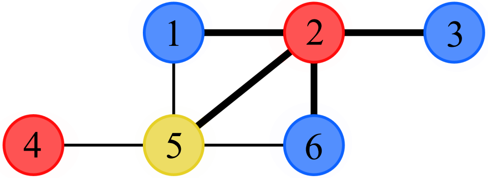
5: **red**

6: **red**

1: **blue**

Conflict-Free Coloring





1: **red**

2: **blue**

3: **blue**

4: **red**

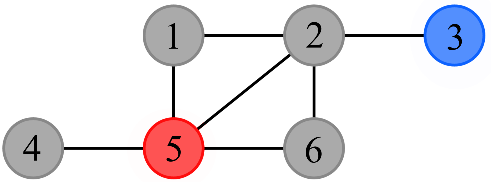
5: **red**

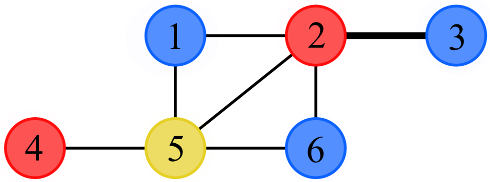
6: **red**

1: **blue**

2: **red**

Conflict-Free Coloring





1: **red**

2: **blue**

3: **blue**

4: **red**

5: **red**

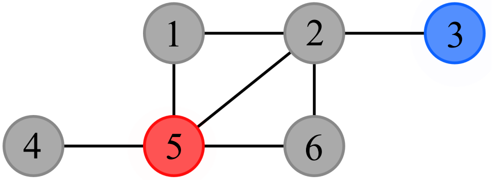
6: **red**

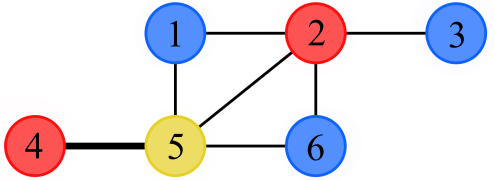
1: **blue**

2: **red**

3: **blue**

Conflict-Free Coloring

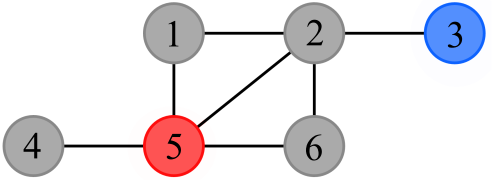


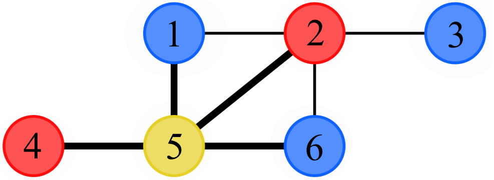


1: **red**
2: **blue**
3: **blue**
4: **red**
5: **red**
6: **red**

1: **blue**
2: **red**
3: **blue**
4: **red**

Conflict-Free Coloring





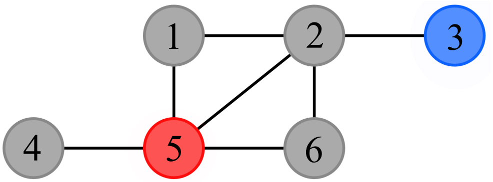
1: **red**
2: **blue**
3: **blue**

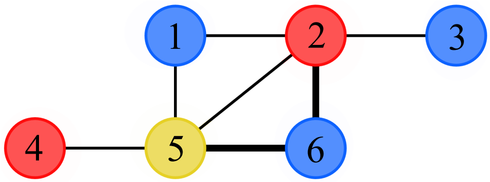
4: **red**
5: **red**
6: **red**

1: **blue**
2: **red**
3: **blue**

4: **red**
5: **yellow**

Conflict-Free Coloring





1: **red**	4: **red**	1: **blue**	4: **red**
2: **blue**	5: **red**	2: **red**	5: **yellow**
3: **blue**	6: **red**	3: **blue**	6: **blue**

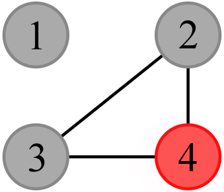
Conflict-Free Coloring Examples

Incorrect conflict-free colorings and their fixes:

Conflict-Free Coloring Examples

Incorrect conflict-free colorings and their fixes:

Isolated vertex



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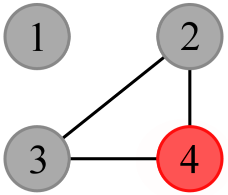

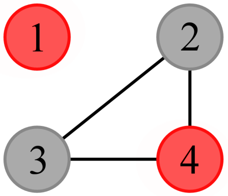

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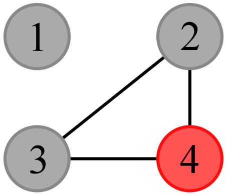
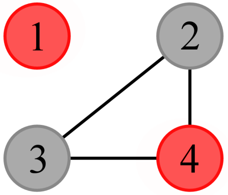
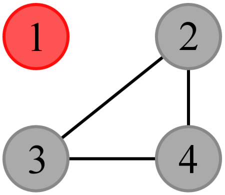




Triangle



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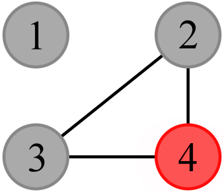

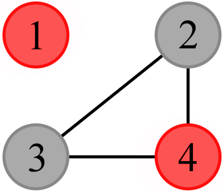

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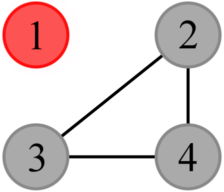

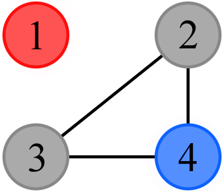

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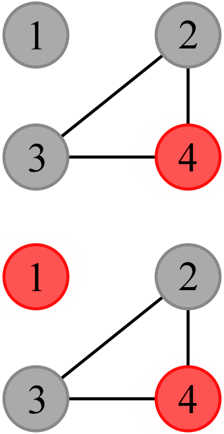
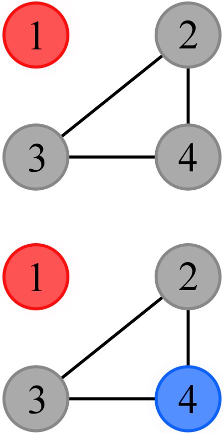
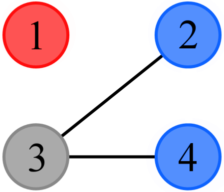


Triangle



Missing unique



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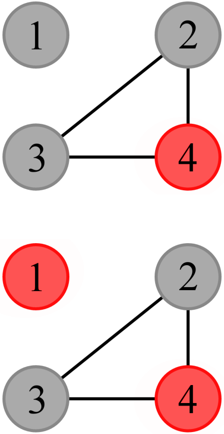

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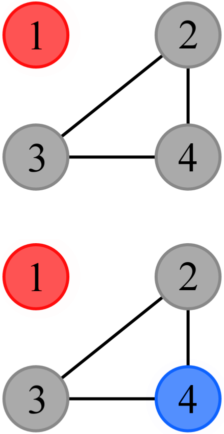

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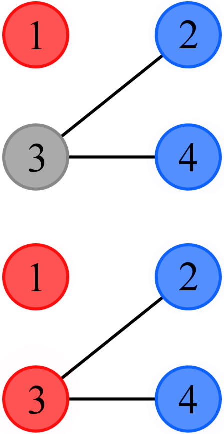

Applications

- **Applications**: wireless networks, satellite communication systems, RFID networks.

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- **Cellular networks**: consist of towers and clients.

Applications

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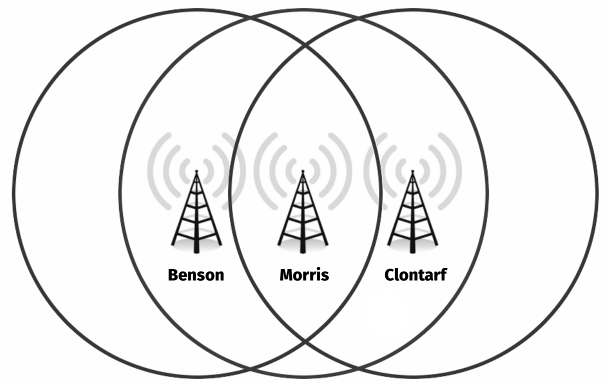
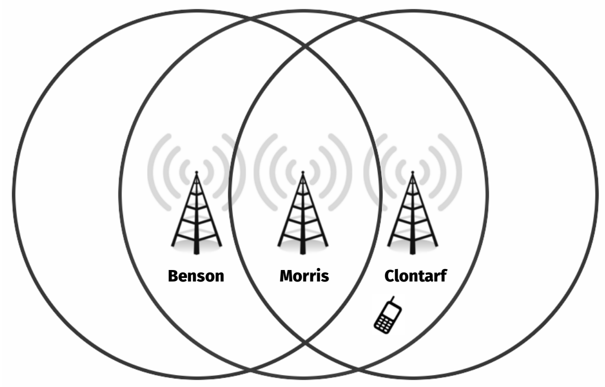
- **Cellular networks**: consist of towers and clients.
- The problem: **frequency assignment**.

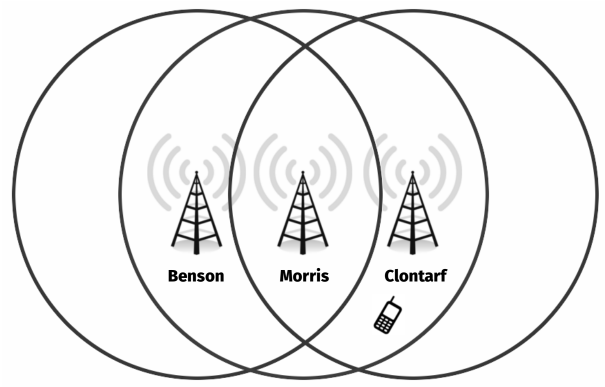
Cellular Networks



Cellular Networks

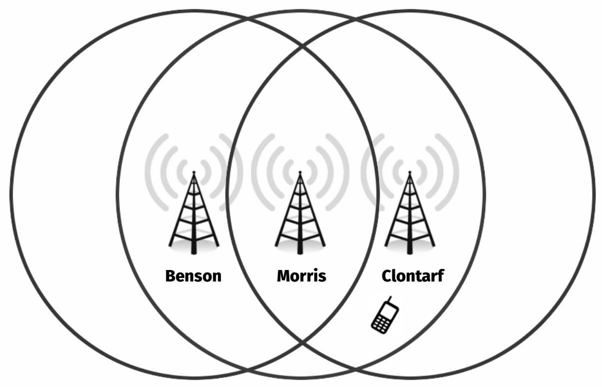


Cellular Networks

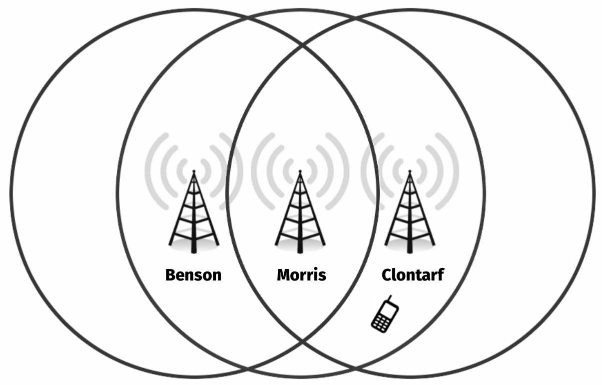


- Our goal is to assign frequencies such that:

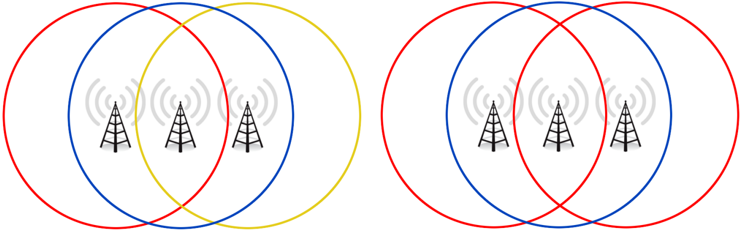
Cellular Networks



- Our goal is to assign frequencies such that:
  (1) Every client is served by a tower with a unique frequency.



- Our goal is to assign frequencies such that:
  (1) Every client is served by a tower with a unique frequency.
  (2) Minimize the number of frequencies used.



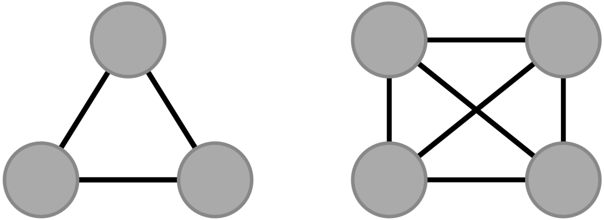
Vertex coloring and conflict-free coloring of cellular towers, respectively

Guaranteeing Conflict-Free k-Colorability

We want to guarantee a graph can be colored with k colors.

Guaranteeing Conflict-Free k-Colorability

We want to guarantee a graph can be colored with *k* colors.

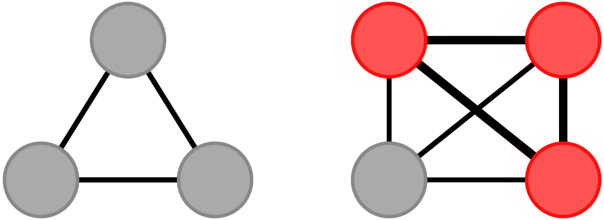


Complete 3 vertex and complete 4 vertex graphs

- **Complete graph**: every pair of distinct vertices is connected by an edge.

Guaranteeing Conflict-Free k-Colorability

We want to guarantee a graph can be colored with k colors.

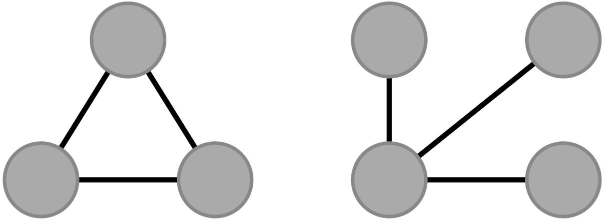


Complete 3 vertex and complete 4 vertex graphs

- **Complete graph**: every pair of distinct vertices is connected by an edge.
- **Tattered graph**: complete graph with a triangle removed.

Guaranteeing Conflict-Free k-Colorability

We want to guarantee a graph can be colored with k colors.

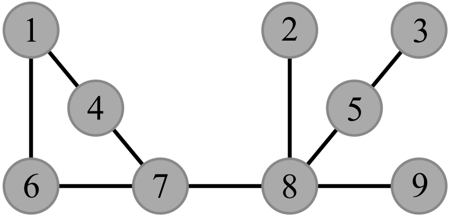


Complete 3 vertex and tattered 4 vertex graphs

- **Complete graph**: every pair of distinct vertices is connected by an edge.
- **Tattered graph**: complete graph with a triangle removed.

Guaranteeing Conflict-Free k-Colorability

Let's demonstrate meeting this criterion on a graph for the simplest case, 1 color.

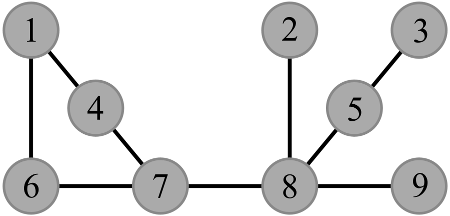


A simple graph

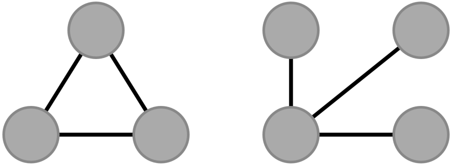
A graph is a **minor** if it can be formed by deleting and/or contracting edges from its parent graph.

Guaranteeing Conflict-Free k-Colorability

Let's demonstrate meeting this criterion on a graph for the simplest case, 1 color.



A simple graph

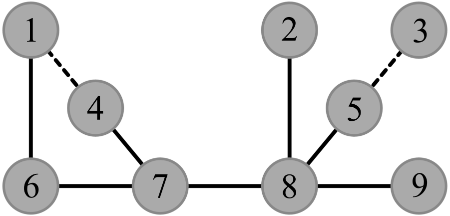


Complete 3 vertex and tattered 4 vertex graphs

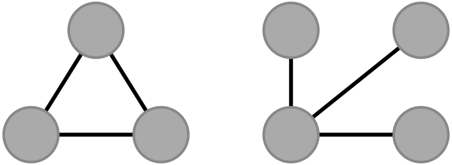
A graph is a **minor** if it can be formed by deleting and/or contracting edges from its parent graph.

Guaranteeing Conflict-Free k-Colorability

Let's demonstrate meeting this criterion on a graph for the simplest case, 1 color.



A simple graph



Complete 3 vertex and tattered 4 vertex graphs

A graph is a **minor** if it can be formed by deleting and/or contracting edges from its parent graph. G contains both graphs as a minor and thus **cannot** be conflict-free colored with 1 color.

Guaranteeing Conflict-Free k-Colorability

Let's demonstrate meeting this criterion on a graph for the simplest case, 1 color.

Theorem

A graph cannot be conflict-free colored with k colors if it contains a complete graph of $k+2$ vertices or a tattered graph of $k+3$ vertices as a minor.

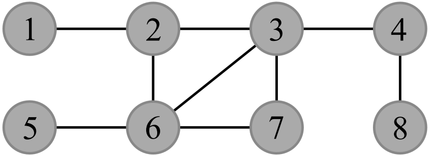
A graph is a **minor** if it can be formed by deleting and/or contracting edges from its parent graph. G contains both graphs as a minor and thus **cannot** be conflict-free colored with 1 color.

Iterated Elimination of Distance-3-Sets

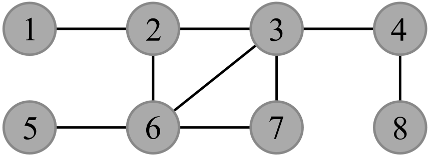
Algorithm IEDS

$i \leftarrow 1,\ P \leftarrow \emptyset$
Remove all isolated paths from G
while G is not empty **do**
    $D \leftarrow \emptyset$
    for all components of G **do**
        Pick any vertex v
        $D \leftarrow D \cup \{v\}$
        while $\exists u$ at distance $\geq 3\ \forall v \in D$ **do**
            Pick u at distance 3 from some vertex in D
            $D \leftarrow D \cup \{w\}$
        for all $u \in D$ **do**
            Color u with color i
        $i \leftarrow i + 1$
        for all $u \in D$ **do**
            Remove $N(u)$ from G
        Remove all isolated paths from G
Color all removed isolated paths using color i

Colors: $\{1 : red,\ 2 : blue\}$



Coloring of G so far



A simple graph G

Iterated Elimination of Distance-3-Sets

Algorithm IEDS

$i \leftarrow 1,\ P \leftarrow \emptyset$
Remove all isolated paths from G
while G is not empty **do**
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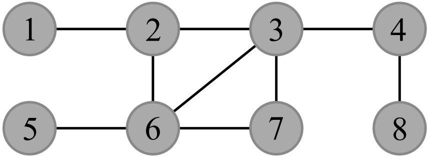
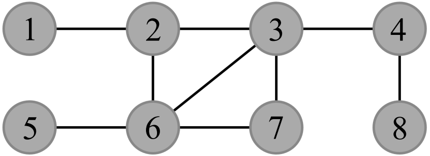
      Remove $N(u)$ from G
    Remove all isolated paths from G
Color all removed isolated paths using color i

Colors: $\{1 : red,\ 2 : blue\}$



Coloring of G so far



A simple graph G

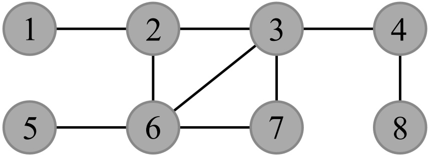
$i = 1$
$P = \{\}$

Iterated Elimination of Distance-3-Sets

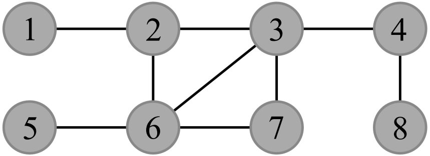
Algorithm IEDS

```
i ← 1,  P ← ∅
Remove all isolated paths from G
while G is not empty do
    D ← ∅
    for all components of G do
        Pick any vertex v
        D ← D ∪ {v}
        while ∃u at distance ≥ 3 ∀v ∈ D do
            Pick u at distance 3 from some vertex in D
            D ← D ∪ {w}
        for all u ∈ D do
            Color u with color i
        i ← i + 1
        for all u ∈ D do
            Remove N(u) from G
        Remove all isolated paths from G
Color all removed isolated paths using color i
```

Colors: $\{1 : red,\ 2 : blue\}$



Coloring of G so far



A simple graph G

$$i = 1$$
$$P = \{\}$$
$$D = \{\}$$

Iterated Elimination of Distance-3-Sets

Algorithm IEDS

$i \leftarrow 1,\ P \leftarrow \emptyset$
Remove all isolated paths from G
while G is not empty **do**
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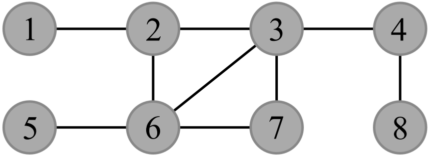
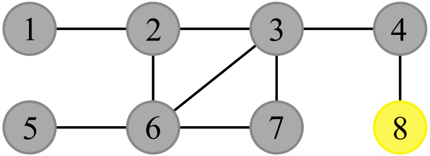
Color all removed isolated paths using color i

Colors: $\{1 : red,\ 2 : blue\}$



Coloring of G so far



A simple graph G

$i = 1$
$P = \{\}$
$D = \{8\}$

Iterated Elimination of Distance-3-Sets

Algorithm IEDS

$i \leftarrow 1,\ P \leftarrow \emptyset$
Remove all isolated paths from G
while G is not empty **do**
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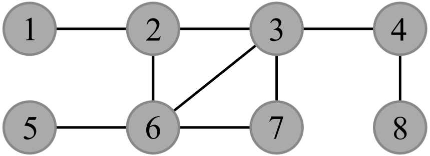
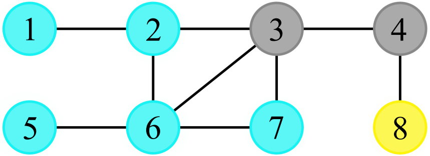
Colors: $\{1 : red,\ 2 : blue\}$



Coloring of G so far



A simple graph G

$i = 1$
$P = \{\}$
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Iterated Elimination of Distance-3-Sets

Algorithm IEDS

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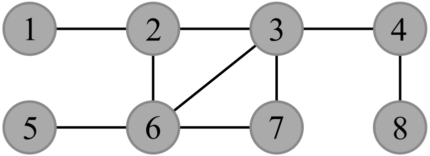
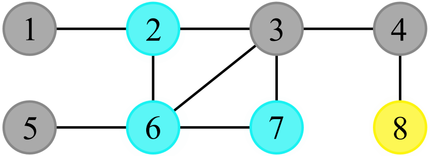
Colors: $\{1 : red,\ 2 : blue\}$



Coloring of G so far



A simple graph G

$i = 1$
$P = \{\}$
$D = \{8\}$

Iterated Elimination of Distance-3-Sets

Algorithm IEDS

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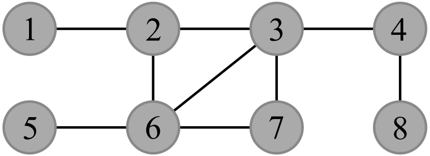
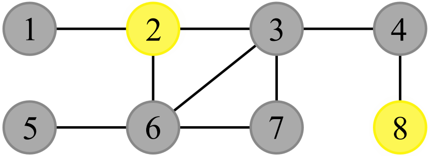


Coloring of G so far



A simple graph G

$i = 1$
$P = \{\}$
$D = \{2, 8\}$

Iterated Elimination of Distance-3-Sets

Algorithm IEDS

$i \leftarrow 1,\ P \leftarrow \emptyset$
Remove all isolated paths from G
while G is not empty **do**
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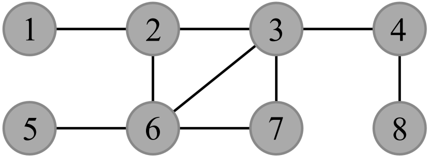
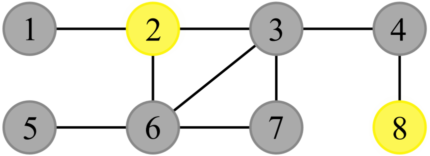


Coloring of G so far



A simple graph G

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$D = \{2, 8\}$

Algorithm IEDS

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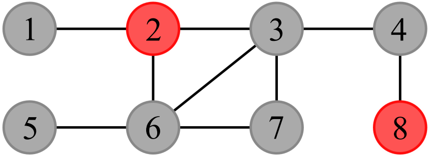
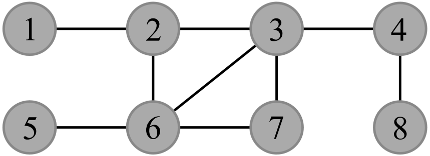


Coloring of G so far



A simple graph G

$$i = 2$$
$$P = \{\}$$
$$D = \{2, 8\}$$

Algorithm IEDS

$i \leftarrow 1,\ P \leftarrow \emptyset$
Remove all isolated paths from G
while G is not empty **do**
  $D \leftarrow \emptyset$
  for all components of G **do**
    Pick any vertex v
    $D \leftarrow D \cup \{v\}$
    while $\exists u$ at distance $\geq 3\ \forall v \in D$ **do**
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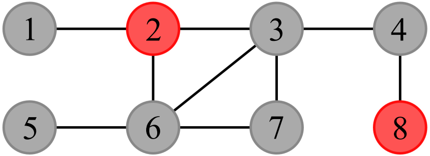
    for all $u \in D$ **do**
      Color u with color i
    $i \leftarrow i + 1$
    for all $u \in D$ **do**
      Remove $N(u)$ from G
    Remove all isolated paths from G
Color all removed isolated paths using color i

Colors: $\{1 : red,\ 2 : blue\}$



Coloring of G so far

A simple graph G

$$i = 2$$
$$P = \{\}$$
$$D = \{2, 8\}$$

Iterated Elimination of Distance-3-Sets

Algorithm IEDS

```
i ← 1,  P ← ∅
Remove all isolated paths from G
while G is not empty do
    D ← ∅
    for all components of G do
        Pick any vertex v
        D ← D ∪ {v}
        while ∃u at distance ≥ 3 ∀v ∈ D do
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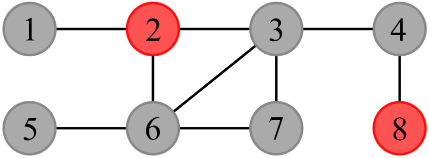
        for all u ∈ D do
            Color u with color i
        i ← i + 1
        for all u ∈ D do
            Remove N(u) from G
        Remove all isolated paths from G
Color all removed isolated paths using color i
```

Colors: $\{1 : red,\ 2 : blue\}$



Coloring of G so far

G is now empty

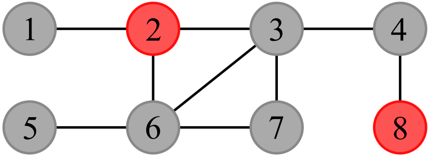
$$i = 2$$
$$P = \{\{5\}, \{7\}\}$$
$$D = \{2, 8\}$$

Iterated Elimination of Distance-3-Sets

Algorithm IEDS

```
i ← 1,  P ← ∅
Remove all isolated paths from G
while G is not empty do
    D ← ∅
    for all components of G do
        Pick any vertex v
        D ← D ∪ {v}
        while ∃u at distance ≥ 3 ∀v ∈ D do
            Pick u at distance 3 from some vertex in D
            D ← D ∪ {w}
        for all u ∈ D do
            Color u with color i
        i ← i + 1
        for all u ∈ D do
            Remove N(u) from G
        Remove all isolated paths from G
Color all removed isolated paths using color i
```

Colors: $\{1 : red,\ 2 : blue\}$



Coloring of G so far

Coloring removed isolated paths

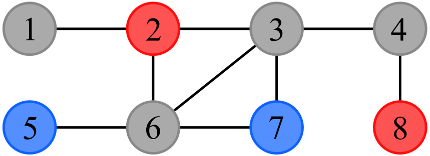
$$i = 2$$
$$P = \{\{5\}, \{7\}\}$$
$$D = \{2, 8\}$$

Iterated Elimination of Distance-3-Sets

Algorithm IEDS

$i \leftarrow 1,\ P \leftarrow \emptyset$
Remove all isolated paths from G
while G is not empty **do**
  $D \leftarrow \emptyset$
  for all components of G **do**
    Pick any vertex v
    $D \leftarrow D \cup \{v\}$
    while $\exists u$ at distance $\geq 3\ \forall v \in D$ **do**
      Pick u at distance 3 from some vertex in D
      $D \leftarrow D \cup \{w\}$
    for all $u \in D$ **do**
      Color u with color i
    $i \leftarrow i + 1$
    for all $u \in D$ **do**
      Remove $N(u)$ from G
    Remove all isolated paths from G
Color all removed isolated paths using color i

Colors: $\{1 : red,\ 2 : blue\}$



Final coloring of G

$$i = 2$$
$$P = \{\{5\}, \{7\}\}$$
$$D = \{2, 8\}$$

Theorem

Every loopless planar graph admits a proper **vertex coloring** *with at most four distinct colors.*

Bounds on Planar Graphs

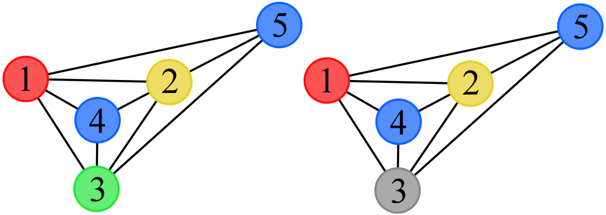
Theorem

Every loopless planar graph admits a proper ***vertex coloring*** *with at most four distinct colors.*

Theorem

Every loopless planar graph admits a ***conflict-free coloring*** *with at most three distinct colors.*

Bounds on Planar Graphs



A vertex and conflict-free coloring, respectively

Theorem

Every loopless planar graph admits a proper ***vertex coloring*** *with at most four distinct colors.*

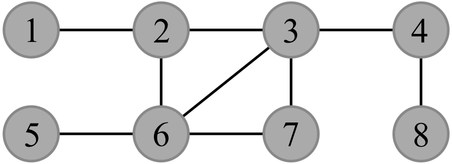
Theorem

Every loopless planar graph admits a ***conflict-free coloring*** *with at most three distinct colors.*

Conflict-Free Coloring via Dominating Set

Algorithm Dominating Set

Find a dominating set, D, of G
for all $v \in V \setminus D$ **do**
    Pick a vertex $u \in D$ where $\{u, v\} \in E$
    Contract the edge $\{u, v\}$ towards u
Find a proper vertex coloring of G
Color the original G with the found coloring

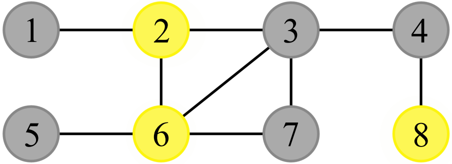


A simple, undirected graph $G = (V, E)$

Conflict-Free Coloring via Dominating Set

Algorithm Dominating Set

1: Find a dominating set, D, of G
2: **for all** $v \in V \setminus D$ **do**
3: Pick a vertex $u \in D$ where $\{u, v\} \in E$
4: Contract the edge $\{u, v\}$ towards u
5: Find a proper vertex coloring of G
6: Color the original G with the found coloring



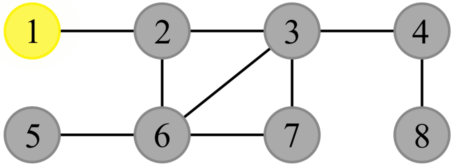
A dominating set of G

$$D = \{2, 6, 8\}$$

Conflict-Free Coloring via Dominating Set

Algorithm Dominating Set

```
Find a dominating set, D, of G
for all v ∈ V \ D do
    Pick a vertex u ∈ D where {u, v} ∈ E
    Contract the edge {u, v} towards u
Find a proper vertex coloring of G
Color the original G with the found coloring
```



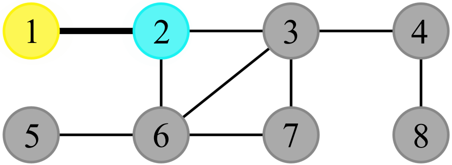
Pick v to be vertex 1

$$D = \{2, 6, 8\}$$
$$V \setminus D = \{1, 3, 4, 5, 7\}$$
$$v \in V \setminus D = 1$$

Conflict-Free Coloring via Dominating Set

Algorithm Dominating Set

Find a dominating set, D, of G
for all $v \in V \setminus D$ **do**
  Pick a vertex $u \in D$ where $\{u, v\} \in E$
  Contract the edge $\{u, v\}$ towards u
Find a proper vertex coloring of G
Color the original G with the found coloring



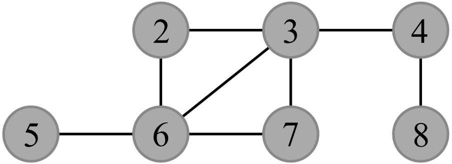
Pick u to be vertex 2

$$D = \{2, 6, 8\}$$
$$V \setminus D = \{1, 3, 4, 5, 7\}$$
$$v \in V \setminus D = 1$$
$$u \in D = 2, \{1, 2\} \in E$$

Conflict-Free Coloring via Dominating Set

Algorithm Dominating Set

1: Find a dominating set, D, of G
2: **for all** $v \in V \setminus D$ **do**
3: Pick a vertex $u \in D$ where $\{u, v\} \in E$
4: Contract the edge $\{u, v\}$ towards u
5: Find a proper vertex coloring of G
6: Color the original G with the found coloring



Contract $\{1, 2\}$ towards 2

$$D = \{2, 6, 8\}$$
$$V \setminus D = \{3, 4, 5, 7\}$$
$$v \in V \setminus D = 1$$

Conflict-Free Coloring via Dominating Set

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for all $v \in V \setminus D$ **do**
    Pick a vertex $u \in D$ where $\{u, v\} \in E$
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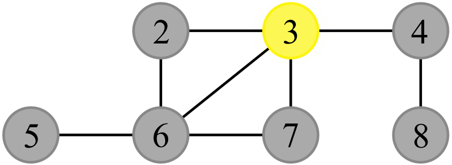
Color the original G with the found coloring



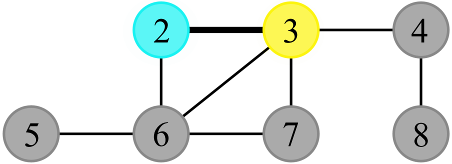
Pick v to be vertex 3

$$D = \{2, 6, 8\}$$
$$V \setminus D = \{3, 4, 5, 7\}$$
$$v \in V \setminus D = 3$$

Conflict-Free Coloring via Dominating Set

Algorithm Dominating Set

Find a dominating set, D, of G
for all $v \in V \setminus D$ **do**
    Pick a vertex $u \in D$ where $\{u, v\} \in E$
    Contract the edge $\{u, v\}$ towards u
Find a proper vertex coloring of G
Color the original G with the found coloring



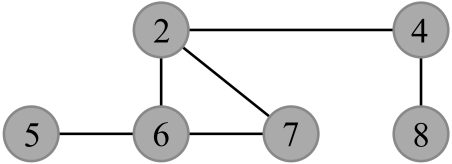
Pick u to be vertex 2

$$D = \{2, 6, 8\}$$
$$V \setminus D = \{3, 4, 5, 7\}$$
$$v \in V \setminus D = 3$$
$$u \in D = 2, \{2, 3\} \in E$$

Conflict-Free Coloring via Dominating Set

Algorithm Dominating Set

1: Find a dominating set, D, of G
2: **for all** $v \in V \setminus D$ **do**
3: Pick a vertex $u \in D$ where $\{u, v\} \in E$
4: Contract the edge $\{u, v\}$ towards u
5: Find a proper vertex coloring of G
6: Color the original G with the found coloring



Contract $\{2, 3\}$ towards 2

$$D = \{2, 6, 8\}$$
$$V \setminus D = \{4, 5, 7\}$$

Conflict-Free Coloring via Dominating Set

Algorithm Dominating Set

1: Find a dominating set, D, of G
2: **for all** $v \in V \setminus D$ **do**
3: Pick a vertex $u \in D$ where $\{u, v\} \in E$
4: Contract the edge $\{u, v\}$ towards u
5: Find a proper vertex coloring of G
6: Color the original G with the found coloring

G after lines 2-4

$$D = \{2, 6, 8\}$$
$$V \setminus D = \{\}$$

Conflict-Free Coloring via Dominating Set

Algorithm Dominating Set

```
Find a dominating set, D, of G
for all v ∈ V \ D do
    Pick a vertex u ∈ D where {u, v} ∈ E
    Contract the edge {u, v} towards u
Find a proper vertex coloring of G
Color the original G with the found coloring
```

A proper vertex coloring

$$D = \{2, 6, 8\}$$
$$V \setminus D = \{\}$$

Conflict-Free Coloring via Dominating Set

Algorithm Dominating Set

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2: **for all** $v \in V \setminus D$ **do**
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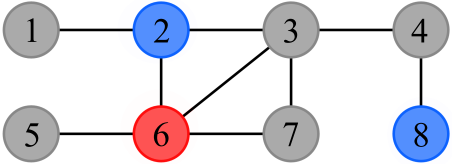

Original G colored

$$D = \{2, 6, 8\}$$
$$V \setminus D = \{\}$$

Conflict-Free Coloring via Dominating Set

Algorithm Dominating Set

```
Find a dominating set, D, of G
for all v ∈ V \ D do
    Pick a vertex u ∈ D where {u, v} ∈ E
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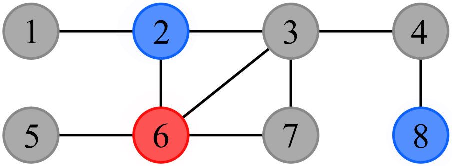

Original G colored

(1) Produces a valid conflict-free coloring

(2) Tries to minimize the number of colored vertices

$$D = \{2, 6, 8\}$$
$$V \setminus D = \{\}$$

Future Work

- Finding bounds and properties on specific graphs such as outerplanar graphs, interval graphs, hypergraphs, and more.

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    - Allows for accurate estimates when applying conflict-free coloring to real-world problems.
- Variations of conflict-free coloring such as requiring **another** vertex to have a unique color within the neighborhood of a selected vertex.

Future Work

- Finding bounds and properties on specific graphs such as outerplanar graphs, interval graphs, hypergraphs, and more.
    - Allows for accurate estimates when applying conflict-free coloring to real-world problems.
- Variations of conflict-free coloring such as requiring **another** vertex to have a unique color within the neighborhood of a selected vertex.
    - Guiding a robot (unique color 1) to a destination (unique color 2).

Thanks to Peter Dolan, Elena Machkasova, and Peh Ng for their advice and feedback.

github.com/devshawn/senior-seminar

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