Filling Gaps on the Pareto Front in Multi- and Many-Objective Optimization

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Introduction: Example

- While designing a school system, you need to balance school construction cost with student transit time
  - You could spend lots of money on schools and have a low travel time
  - You could spend less money to build one school and have high travel time
Introduction: Pareto Front

- Two-objectives:
  - X-axis is the construction cost for schools
  - Y-axis is the average student travel time
- Red points are the Pareto front
  - The non-dominated (best) points for a problem
  - Either higher or to the right of non-Pareto front points
Introduction: Big Questions

- You want *diversification*
  - Wide variety of points
- How do you make sure you found all of the points?
- What if there’s a hole in the Pareto front?
Outline

1. Background
   a. Quick Graph Legend
   b. Gaps on the Pareto Front

2. Detecting Gaps (Pellicer et al)
   a. Multi-Objective Evolutionary Algorithm
   b. Many-Objective Evolutionary Algorithm
   c. Find Potential Gaps
   d. Validate the Gaps

3. Filling Gaps (Pellicer et al)
   a. Focused Multi-Objective Evolutionary Algorithm
   b. Focused Many-Objective Evolutionary Algorithm
   c. Refresher on Gaps
   d. Classifying Gaps

4. Results (Pellicer et al)
   a. Legend for Graphs
   b. ZDT3
   c. DTLZ2
   d. Steel Industry
   e. Slung-Load Quadrotor (Ergezer)

5. Conclusion
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Quick Graph Legend

- Grey is original Pareto front
- Black “x” is/are the gap point(s)
- Blue is a focused search Pareto front
- Red is/are closest point(s) to gap point(s)
Gaps on the Pareto Front

- An empty space between two groups of points on a graph
- Can be deemed *natural* or *artificial* once it is *validated*
  - *Validated* means the gap is an actual gap
  - *Natural* implies no point can exist in that space
  - *Artificial* means the gap is considered filled
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Non-dominated Sorting Genetic Algorithm II (NSGA-II)

- Handles multi-objective optimization (2-3 objectives)
- Implements a better system for diversification
- Improved the computational complexity of evolutionary algorithms at the time
Non-dominated Sorting Genetic Algorithm III (NSGA-III)

- Builds off of NSGA-II to handle many-objective optimization (3+ objectives)
- Uses an additional method of reference points to improve diversification
  - Reference points consider points closer to it better
Finding Gaps

- Created an algorithm to find and validate gaps
- Organize Pareto front into groups
- Find the most isolated cluster and validate the gap
- If the gap is validated, save it and add a new cluster for that gap
Validating Gaps

- Take every point found through the algorithm and run equation 1
  - $K$ is number of gap points
  - $N$ is Pareto front size
- $d_{G-EMO} > 1$ means the gap is validated
- $d_{G-EMO} < 1$ means the gap isn’t real

$$d_{G-EMO} = \frac{\frac{1}{K} \sum_{i=1}^{K} (\min_{j=1}^{N} \|G_i - F_j\|_2)}{\frac{1}{N} \sum_{i=1}^{N} (\min_{j=1}^{N} \|F_i - F_j\|_2)}$$ (1)
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Reference point-based Non-dominated Sorting Genetic Algorithm II (R-NSGA-II)

- Useful for multi-objective problems (2-3 objectives)
- Utilizes the reference points from NSGA-III to target a specific area on the Pareto front
Reference point-based Non-dominated Sorting Genetic Algorithm III (R-NSGA-III)

- Used to target a specific area in a many-objective problem (3+ objectives)
- Uses an additional method to ensure even distribution among reference points, making results more diverse
Refresher: Gaps on the Pareto Front

- Empty space between two groups of points
- Can be deemed *natural* or *artificial*
  - *Natural* implies no point can exist in that space
  - *Artificial* means the gap is considered filled
Classifying Gaps

- Take every validated gap point and run equation 2
  - \( K \) is number of gap points
  - \( N \) is Pareto front size
  - \( R \) is number of R-NSGA points
- \( d_{R-G} > 1 \) means the gap is natural
- \( d_{R-G} < 1 \) means the gap is artificial
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Legend for Graphs

- Grey is original Pareto front (NSGA-II(I))
- Black “x” is/are the gap point(s)
- Blue is new Pareto front (R-NSGA-II(I))
- Red is/are closest point(s) to gap point(s)
ZDT3 Problem

- Two-objective test problem
- Utilizes NSGA-II to generate its Pareto front and R-NSGA-II to fill gaps
- Has a discontinuous graph with visual holes
- $f_1$ and $f_2$ represent the two objectives being optimized
- Ran with one gap point and three gap points
ZDT3 Results

- One gap point
  - $d_{G-EMO} = 2.876$, meaning the gap is validated
  - $d_{R-G} = 1969.522$, meaning the gap is natural
- Three gap points
  - $d_{G-EMO} = 5.316$, meaning the gaps are validated
  - $d_{R-G} = 35.897$, meaning the gaps are natural
DTLZ2 Problem

- Three-objective test problem
- Utilizes NSGA-III to generate its Pareto front and R-NSGA-III to fill gaps
- Has a spherical shape for its Pareto front
- Two variations: DTLZ2\text{orig} and DTLZ2\text{void}
  - DTLZ2\text{orig} is the full Pareto front
  - DTLZ2\text{void} is incomplete
- f1, f2, and f3 represent the three objectives to optimize
- Ran with one gap point and five gap points
DTLZ2$^{\text{orig}}$ Results

- One gap point
  - Algorithm was run three times
  - $d_{G-EMO} = 0.0006, 0.604, \text{ and } 0.540$, meaning the gap is not valid
  - R-NSGA-III was never run
DTLZ2\textsuperscript{void} Results

- One gap point
  - $d_{G-EMO} = 1.330$, meaning the gap is validated
  - $d_{R-G} = 0.680$, meaning the gap is artificial and got filled

- Five gap points
  - $d_{G-EMO} = 1.124$, meaning the gaps are validated
  - $d_{R-G} = 0.572$, meaning the gaps are artificial and got filled
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Steel Industry Problem

- A real-world example of a five-objective test problem
  - A coil factory transforming sheets of steel into coils
- Looks to optimize:
  - The time to complete the order
  - Importance of the project being completed at that given moment
  - Production cost
  - Maintenance expenses
  - Scheduling constraints
- Known to have gaps in its Pareto Front
- Ran with one gap point and three gap points
Steel Industry Results

- One gap point
  - $d_{G-EMO} = 2.302$, meaning the gap is validated
  - $d_{R-G} = 1.442$, meaning the gap is natural

- Three gap points
  - $d_{G-EMO} = 2.430$, meaning the gaps are validated
  - $d_{R-G} = 1.688$, meaning the gaps are natural
Slung-Load Quadrotoor Problem (Ergezer)

- Three-objective problem in the real-world
  - A drone holding a package underneath with a rope
  - Trying to avoid obstacles
- Looks to optimize:
  - Minimize path length
  - Minimize time
  - Conserve energy
- Optimizes two objectives at a time
- Uses NSGA-II and R-NSGA-II
Results from Ergezer

- Scenario 1 (Path Length vs Energy):
  - $d_{R-G} = 1.151$, meaning the gaps are natural
- Scenario 2 (Energy vs Time):
  - $d_{R-G} = 1.782$, meaning the gaps are natural
Conclusion

- Finding and validating gap points allows R-NSGA algorithms to search for additional points and therefore have the potential to find more solutions.
- Having a wider variety of solutions gives us more to choose from.
- Real-world uses:
  - Steel industry example
  - Quadrotor example
  - Decision-makers
Questions?
References
