Filling Gaps on the Pareto Front in Multi- and Many-Objective Optimization

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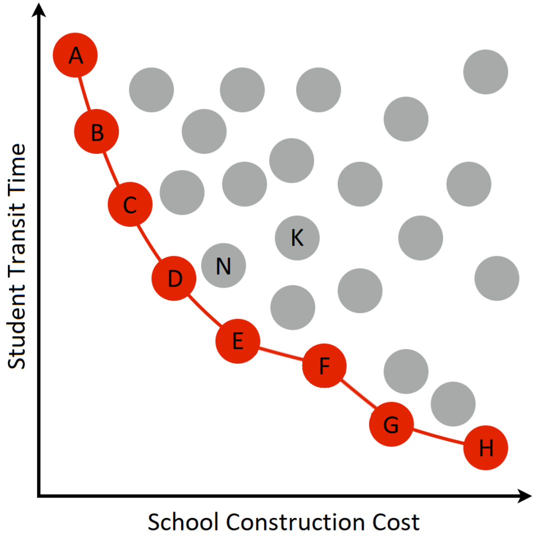
Introduction: Example

- While designing a school system, you need to balance school construction cost with student transit time
  - You could spend lots of money on schools and have a low travel time
  - You could spend less money to build one school and have high travel time

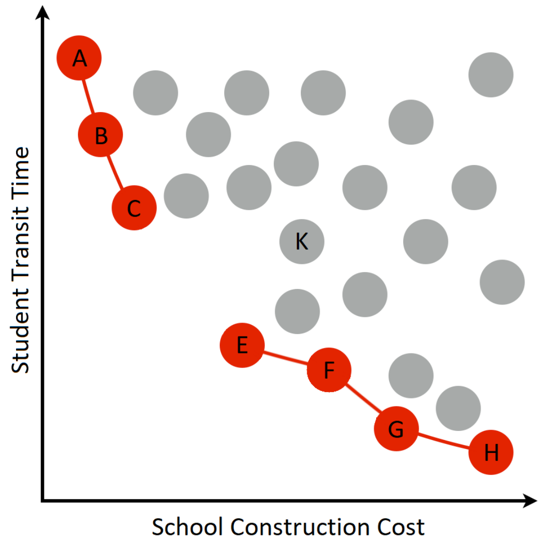
Introduction: Pareto Front

- Two-objectives:
  - X-axis is the construction cost for schools
  - Y-axis is the average student travel time
- Red points are the *Pareto front*
  - The non-dominated (best) points for a problem
  - Either higher or to the right of non-Pareto front points



Introduction: Big Questions

- You want *diversification*
  - Wide variety of points
- How do you make sure you found all of the points?
- What if there's a hole in the Pareto front?



Outline

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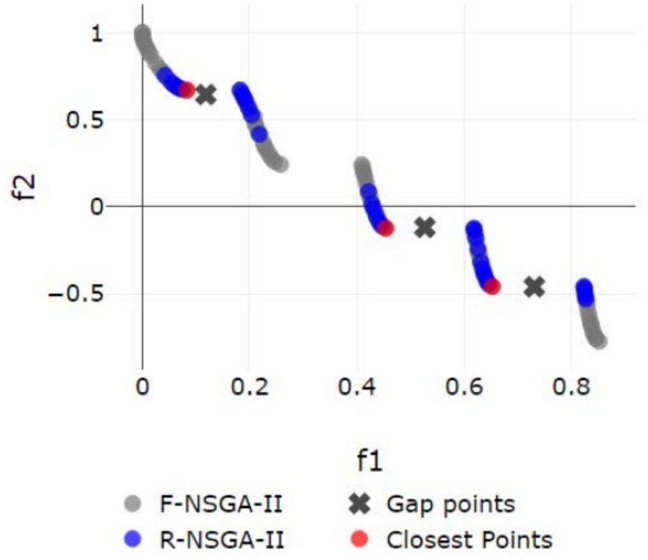
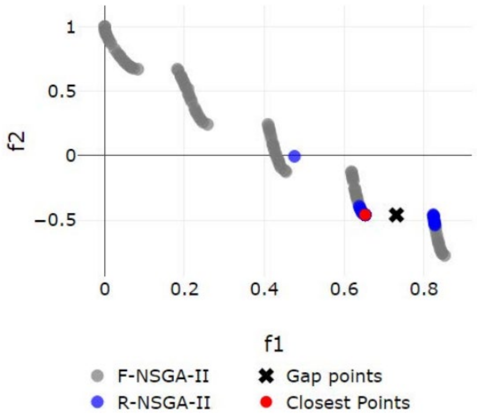
Quick Graph Legend

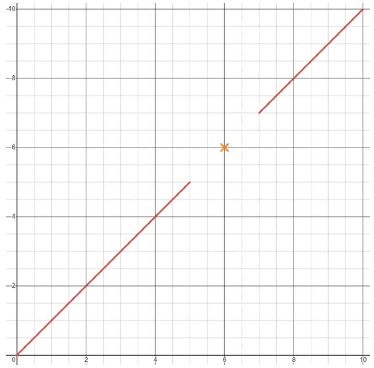
- Grey is original Pareto front
- Black “x” is/are the gap point(s)
- Blue is a focused search Pareto front
- Red is/are closest point(s) to gap point(s)



Gaps on the Pareto Front

- An empty space between two groups of points on a graph
- Can be deemed *natural* or *artificial* once it is *validated*
    - *Validated* means the gap is an actual gap
    - *Natural* implies no point can exist in that space
    - *Artificial* means the gap is considered filled





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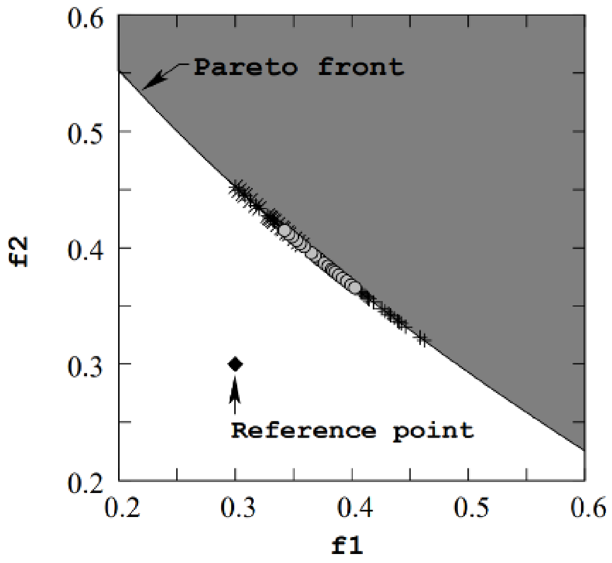
Non-dominated Sorting Genetic Algorithm II (NSGA-II)

- Handles multi-objective optimization (2-3 objectives)
- Implements a better system for diversification
- Improved the computational complexity of evolutionary algorithms at the time

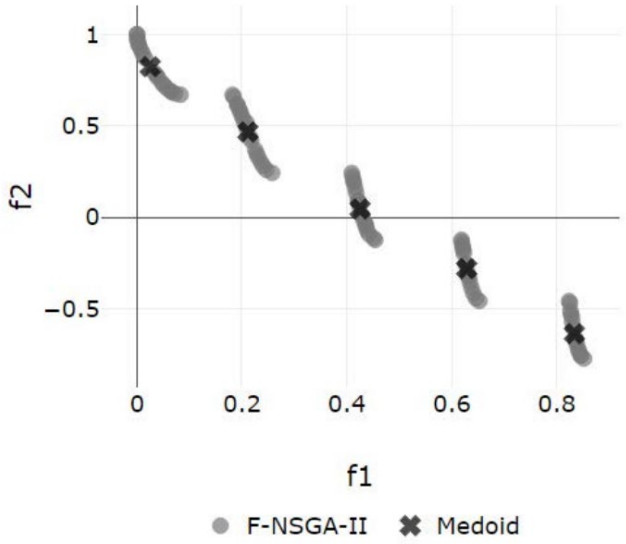
Non-dominated Sorting Genetic Algorithm III (NSGA-III)

- Builds off of NSGA-II to handle many-objective optimization (3+ objectives)
- Uses an additional method of reference points to improve diversification
  - Reference points consider points closer to it better



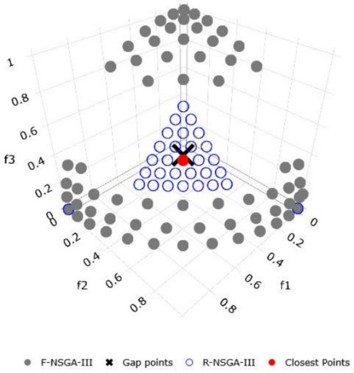
Finding Gaps

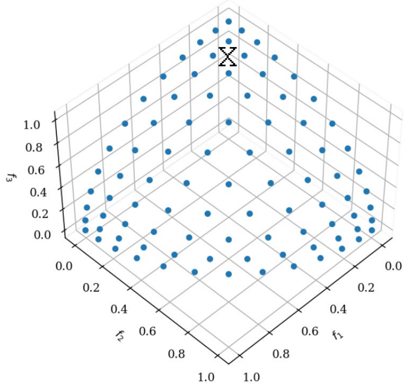
- Created an algorithm to find and validate gaps
- Organize Pareto front into groups
- Find the most isolated cluster and *validate* the gap
- If the gap is validated, save it and add a new cluster for that gap



Validating Gaps

- Take every point found through the algorithm and run equation 1
  - K is number of gap points
  - N is Pareto front size
- $d_{G\text{-}EMO} > 1$ means the gap is *validated*
- $d_{G\text{-}EMO} < 1$ means the gap isn't real





$$d_{G-EMO} = \frac{\frac{1}{K}\sum_{i=i}^{K}(\min_{j=1}^{N} \|G_i - F_j\|_2)}{\frac{1}{N}\sum_{i=1}^{N}(\min_{j=1}^{N} \|F_i - F_j\|_2)} \qquad (1)$$

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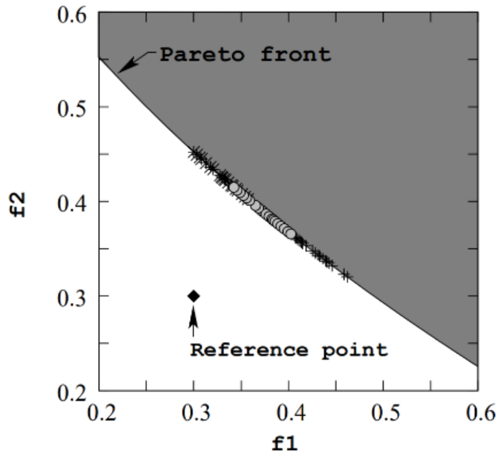
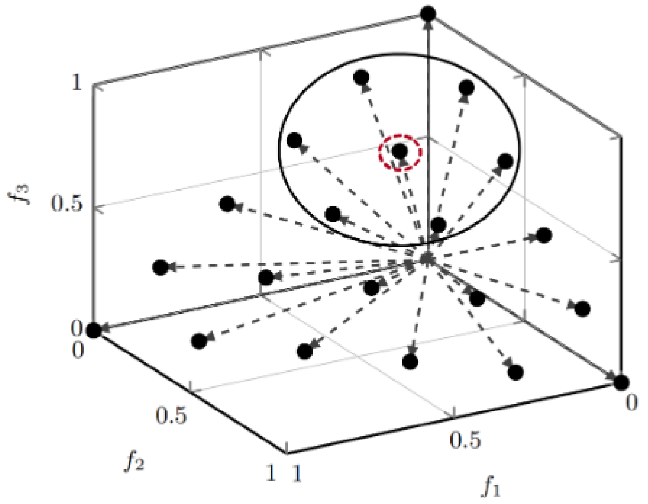
Reference point-based Non-dominated Sorting Genetic Algorithm II (R-NSGA-II)

- Useful for multi-objective problems (2-3 objectives)
- Utilizes the reference points from NSGA-III to target a specific area on the Pareto front



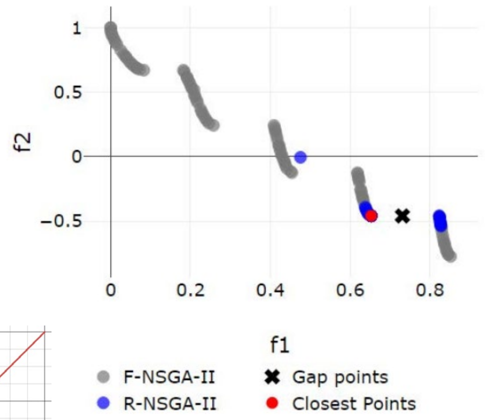
Reference point-based Non-dominated Sorting Genetic Algorithm III (R-NSGA-III)

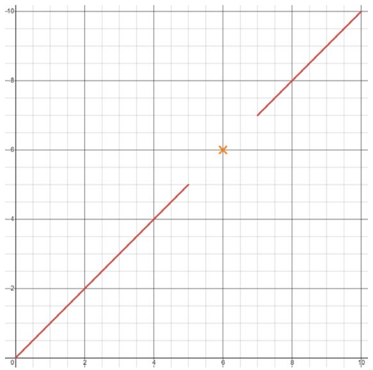
- Used to target a specific area in a many-objective problem (3+ objectives)
- Uses an additional method to ensure even distribution among reference points, making results more diverse



Refresher: Gaps on the Pareto Front

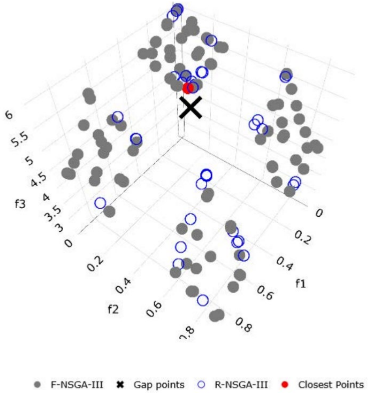
- Empty space between two groups of points
- Can be deemed *natural* or *artificial*
    - *Natural* implies no point can exist in that space
    - *Artificial* means the gap is considered filled

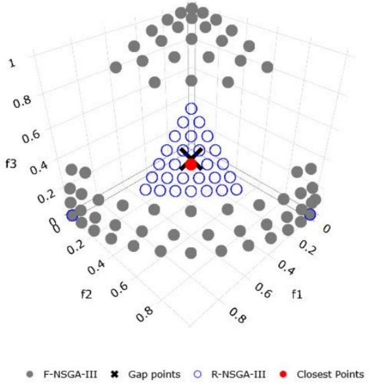




Classifying Gaps

- Take every validated gap point and run equation 2
    - K is number of gap points
    - N is Pareto front size
    - R is number of R-NSGA points
- $d_{R\text{-}G} > 1$ means the gap is *natural*
- $d_{R\text{-}G} < 1$ means the gap is *artificial*



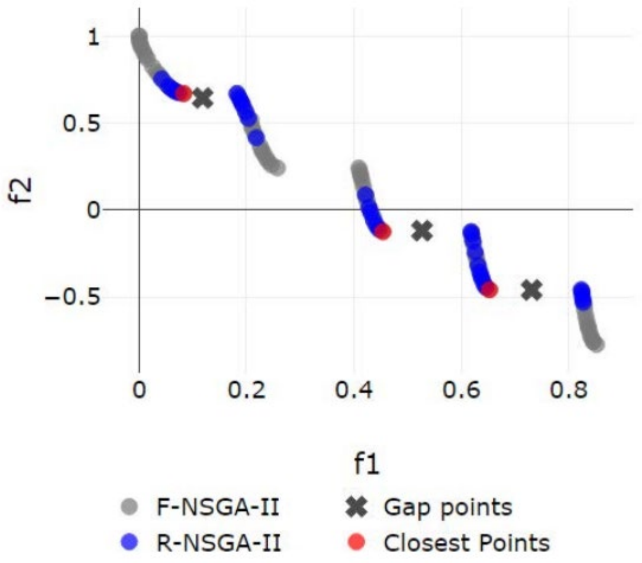


$$d_{R-G} = \frac{1}{R} \sum_{i=1}^{R} \frac{\min_{j=1}^{K} \|RF_i - G_j\|_2}{\min_{j=1}^{N} \|RF_i - F_j\|_2} \qquad (2)$$

Outline

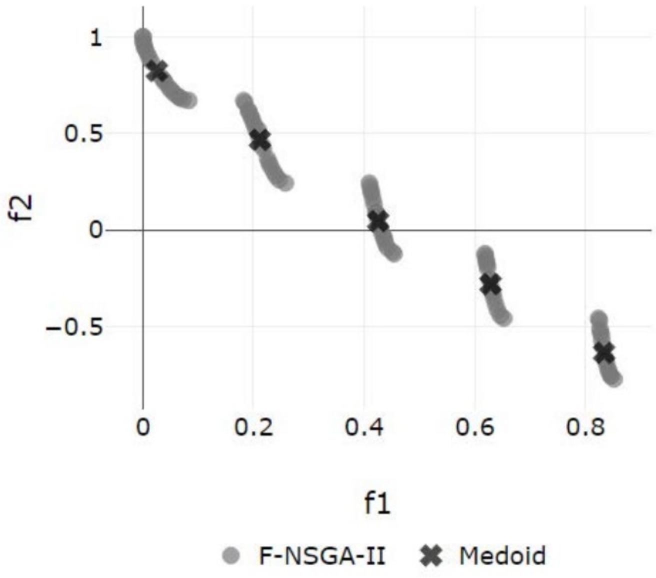
Legend for Graphs

- Grey is original Pareto front (NSGA-II(I))
- Black “x” is/are the gap point(s)
- Blue is new Pareto front (R-NSGA-II(I))
- Red is/are closest point(s) to gap point(s)



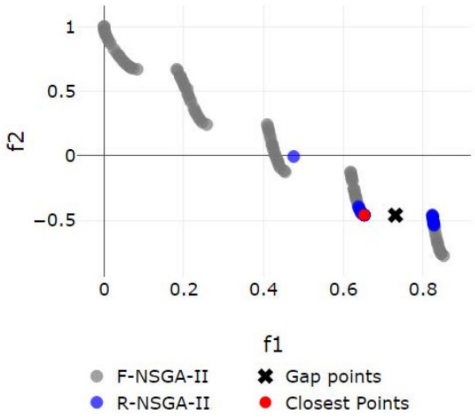
ZDT3 Problem

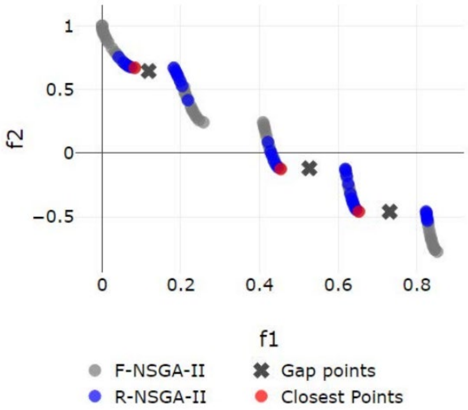
- Two-objective test problem
- Utilizes NSGA-II to generate its Pareto front and R-NSGA-II to fill gaps
- Has a discontinuous graph with visual holes
- f1 and f2 represent the two objectives being optimized
- Ran with one gap point and three gap points



ZDT3 Results

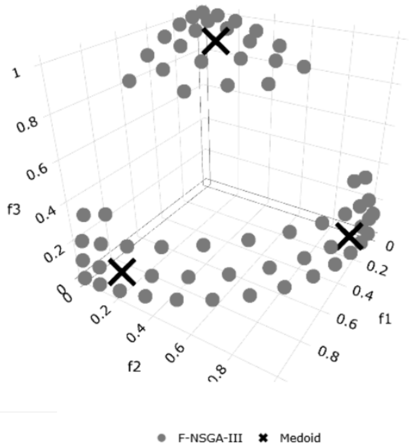
- One gap point
  - $d_{G\text{-}EMO}$ = 2.876, meaning the gap is validated
  - $d_{R\text{-}G}$ = 1969.522, meaning the gap is natural
- Three gap points
  - $d_{G\text{-}EMO}$ = 5.316, meaning the gaps are validated
  - $d_{R\text{-}G}$ = 35.897, meaning the gaps are natural

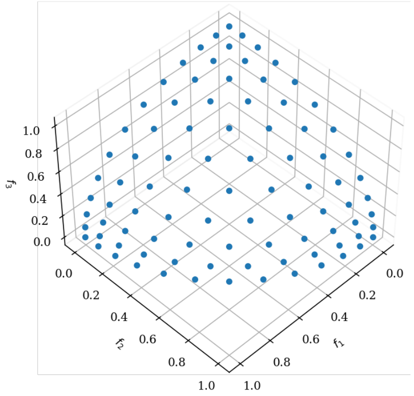




DTLZ2 Problem

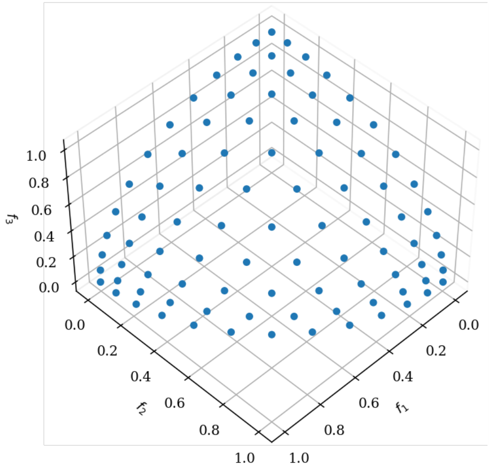
- Three-objective test problem
- Utilizes NSGA-III to generate its Pareto front and R-NSGA-III to fill gaps
- Has a spherical shape for its Pareto front
- Two variations: DTLZ2^{orig} and DTLZ2^{void}
  - DTLZ2^{orig} is the full Pareto front
  - DTLZ2^{void} is incomplete
- f1, f2, and f3 represent the three objectives to optimize
- Ran with one gap point and five gap points





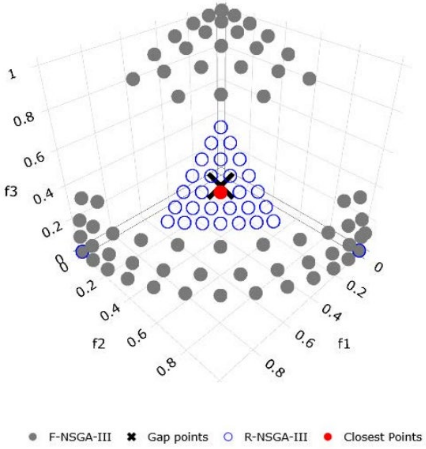
DTLZ2^{orig} Results

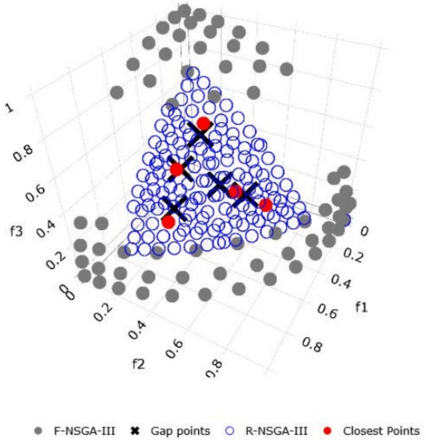
- One gap point
  - Algorithm was run three times
  - $d_{G\text{-}EMO}$ = 0.0006, 0.604, and 0.540, meaning the gap is not valid
  - R-NSGA-III was never run



DTLZ2^{void} Results

- One gap point
  - $d_{G\text{-}EMO}$ = 1.330, meaning the gap is validated
  - $d_{R\text{-}G}$ = 0.680, meaning the gap is artificial and got filled
- Five gap points
  - $d_{G\text{-}EMO}$ = 1.124, meaning the gaps are validated
  - $d_{R\text{-}G}$ = 0.572, meaning the gaps are artificial and got filled





Outline

Steel Industry Problem

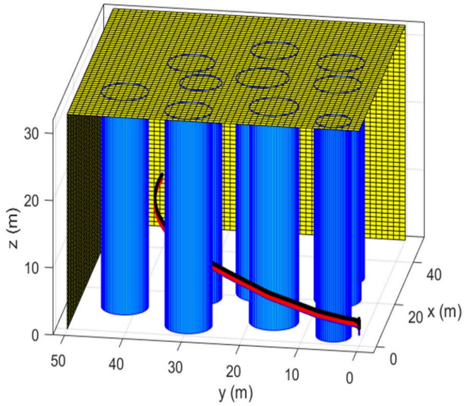
- A real-world example of a five-objective test problem
  - A coil factory transforming sheets of steel into coils
- Looks to optimize:
  - The time to complete the order
  - Importance of the project being completed at that given moment
  - Production cost
  - Maintenance expenses
  - Scheduling constraints
- Known to have gaps in its Pareto Front
- Ran with one gap point and three gap points

Steel Industry Results

- One gap point
  - $d_{G\text{-}EMO}$ = 2.302, meaning the gap is validated
  - $d_{R\text{-}G}$ = 1.442, meaning the gap is natural
- Three gap points
  - $d_{G\text{-}EMO}$ = 2.430, meaning the gaps are validated
  - $d_{R\text{-}G}$ = 1.688, meaning the gaps are natural

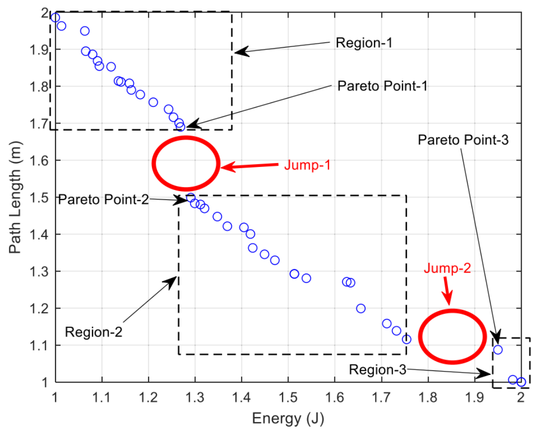
Slung-Load Quadrotor Problem (Ergezer)

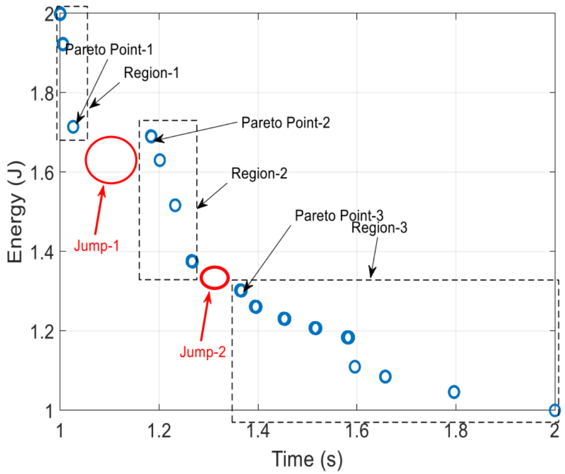
- Three-objective problem in the real-world
  - A drone holding a package underneath with a rope
  - Trying to avoid obstacles
- Looks to optimize:
  - Minimize path length
  - Minimize time
  - Conserve energy
- Optimizes two objectives at a time
- Uses NSGA-II and R-NSGA-II



Results from Ergezer

- Scenario 1 (Path Length vs Energy):
  - $d_{R\text{-}G}$ = 1.151, meaning the gaps are natural
- Scenario 2 (Energy vs Time):
  - $d_{R\text{-}G}$ = 1.782, meaning the gaps are natural





Conclusion

- Finding and validating gap points allows R-NSGA algorithms to search for additional points and therefore have the potential to find more solutions
- Having a wider variety of solutions gives us more to choose from
- Real-world uses:
  - Steel industry example
  - Quadrotor example
  - Decision-makers

Questions?

References

Pablo Valledor Pellicer, Miguel Iglesias Escudero, Silvino Fernández Alzueta, and Kalyanmoy Deb. 2020. Gap finding and validation in evolutionary multi- and many-objective optimization. In Proceedings of the 2020 Genetic and Evolutionary Computation Conference (GECCO '20). Association for Computing Machinery, New York, NY, USA, 578–586. DOI:https://doi-org.ezproxy.morris.umn.edu/10.1145/3377930.3389835

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